TRANSPORT PROBLEMS 2025 Volume 20 Issue 3
PROBLEMY TRANSPORTU DOI: 10.20858/tp.2025.20.3.04

Keywords: smart mini parking systems; smart city; stochastic Petri Nets; modeling and simulation, TimeNET®

Ilija HRISTOSKI[1], Marija MALENKOVSKA TODOROVA[2]*

MODELING AND EVALUATION OF SMART MINI PARKING SYSTEMS WITH PRIOR RESERVATION USING STOCHASTIC PETRI NETS

Summary. Rapid urbanization has significantly increased the demand for efficient parking solutions, particularly in city centers, making urban parking management a critical challenge. Inefficient urban parking management contributes directly to traffic congestion, increased fuel consumption, and environmental pollution. This paper evaluates the modeling and performance of a smart mini parking system that enables prior parking slot reservations through a mobile application. The system comprises seven parking spaces, allocated among four vehicle types as follows: one for handicapped vehicles, three for regular vehicles, one for small utility/cargo vehicles, and two for electric vehicles. A deterministic and stochastic Petri net model is developed to represent the dynamic reservation process, vehicle arrivals and departures, and parking durations. This approach captures the stochastic nature of vehicle flows while incorporating deterministic time constraints for fixed billing intervals. Using numerical simulations, the study evaluates the utilization rates of individual parking slots and the overall system under different vehicle arrival rates and a predefined operating profile reflecting vehicle type distribution. The simulation-based analysis highlights how arrival patterns and fleet composition affect occupancy, system saturation, and service efficiency. The findings indicate the prospective significance of smart mini parking systems, offer practical insights into optimizing parking allocation and reservation strategies in smart urban environments, and demonstrate the potential of deterministic and stochastic Petri nets as a formal tool for modeling and improving smart parking systems, thus contributing to more sustainable urban mobility solutions.

1. INTRODUCTION

The smart city concept was developed as a potential solution and systematic approach to urban development, aimed at improving residents' quality of life. Its origins can be traced back to the period when the sustainability of urban living became uncertain due to rapid urban expansion and its associated consequences. It is a multifaceted concept encompassing several key domains, including transportation, citizen engagement, infrastructure, technology, energy, and urban management [1]. The integration of advanced technologies (e.g., Big Data, cloud computing, Internet of Things) across these key domains, combined with sustainable transport infrastructure, enhances accessibility, safety, security, energy efficiency, and environmental protection [2].

Urban parking management, as a substantial aspect of smart cities, has become a critical challenge due to increasing vehicle ownership and limited parking infrastructure. Despite the hurdles imposed by the severe health and geopolitical crises since 2021, which significantly disrupted supply chains and constrained automotive semiconductor supply, international car sales in 2023 and 2024 exceeded pre-pandemic levels and are expected to continue rising through 2025. Global new car sales rose from

[1] "St. Kliment Ohridski" University – Bitola, Faculty of Economics – Prilep; 143 Prilepski Braniteli St., 7500 Prilep, North Macedonia; e-mail: ilija.hristoski@uklo.edu.mk; orcid.org/0000-0002-9291-7873
[2] "St. Kliment Ohridski" University – Bitola, Faculty of Technical Sciences – Bitola; 37 Makedonska Falanga St., 7000 Bitola, North Macedonia; e-mail: marija.malenkovska@uklo.edu.mk; orcid.org/0000-0001-5620-174X
* Corresponding author. E-mail: marija.malenkovska@uklo.edu.mk

about 75.3 million in 2023 to roughly 78 million in 2024 [3]. In North America, 19.2 million new light automobiles were sold in 2024, with U.S. sales projected to reach 15.9 million in 2025, up from 15.5 million in 2024 [4]. This growth fuels rising private car ownership, intensifying traffic congestion and infrastructure strain. In the U.S., registered vehicles exceeded 283 million in 2022, up nearly 2.5% in three years [5]. Similar trends appear worldwide: Germany hit a record 49.1 million vehicles in early 2024 (580 per 1,000 people) [6], while Monaco and San Marino reported some of the world's highest ownership rates, 899 and 1,263 cars per 1,000 people, respectively. It is estimated that about 95% of U.S. households own a car, and 85% of Americans commute to work by car [7]. Moreover, urban centres worldwide face heavy traffic congestion from the daily influx of light vehicles (mainly passenger cars), during peak commuting hours due to work-related travel [8]. Tourist traffic further worsens the situation, as more travellers use private vehicles, amplifying the negative effects of *overtourism* [9].

Inefficient parking allocation contributes to traffic congestion, increased fuel consumption, and higher carbon emissions as drivers spend significant time searching for available spaces. Traditional parking systems, which rely on first-come-first-served policies, often fail to adapt to real-time demand fluctuations, leading to inefficient space utilization, as well as unnecessary delays, increased carbon emissions, and environmental pollution.

The steadily increasing number of vehicles, particularly passenger cars, in cities and towns worldwide has both created and intensified the ongoing challenge of continually building new parking facilities. To address these issues, smart parking systems have emerged as a promising solution by integrating real-time monitoring, automated space allocation, and reservation mechanisms. The concept of intelligent parking systems, based on secured wireless networks and sensor communication, along with their corresponding hardware/software architectures, implementations, as well as analytical models and simulation results, was proposed more than 15 years ago [10-11]. As a key element in the development of smart cities, smart parking systems, which rely heavily on advanced information and communication technologies, streamline parking operations, reduce time spent searching for parking, and help mitigate severe traffic congestion. However, as cities and towns become increasingly overpopulated, it is foreseeable that available space for constructing large parking facilities within urban areas will soon become scarce. Instead, it is more likely that the concept of small (i.e., mini) smart parking facilities will prevail, providing numerous distributed parking areas across cities, each offering a limited number of parking slots.

Based on the previous observations, this study was conducted to address the growing spatial constraints in overpopulated urban areas, where the construction of large-scale parking facilities is becoming increasingly impractical due to limited land availability, high development costs, and regulatory challenges. Consequently, the focus has shifted toward the implementation of small, distributed smart parking systems as a more scalable, adaptive, and context-sensitive solution.

The primary objective of this study is to evaluate the feasibility and behavior of a hypothetical smart mini parking system through modeling and simulation using the class of deterministic and stochastic Petri nets (DSPNs), which, as a superset of Petri nets (PNs), have proven highly effective in representing and analyzing complex, dynamic systems, thus providing a powerful formalism to model and assess mini parking systems that involve stochastic vehicle arrivals and parking durations, simultaneous reservations, and deterministic billing intervals.

The key contributions of this research are as follows: (a) development of a DSPN-based formal model for dynamic smart parking reservation, integrating both pre-reservation and real-time booking; (b) categorization of parking slots based on vehicle type, parking duration, and special needs; (c) performance evaluation of parking efficiency using key metrics such as average parking slots' utilization rates, average reservation rates per unit of time, and average queue lengths of vehicles searching for a free parking slot and those exiting the parking facility; and (d) simulation-based analysis demonstrating the effectiveness of the proposed model in optimizing urban parking management.

In this context, this study aims to guide the development and evaluation of the proposed DSPN model for dynamic smart parking reservation by addressing the following key research questions:

- RQ1: How can a smart mini parking system with heterogeneous vehicle types and reservation capabilities be effectively modeled using DSPNs?
- RQ2: What impacts do different vehicle arrival rates and a specified operating profile have on the utilization of individual parking slots and the system as a whole?
- RQ3: To what extent do the average reservation rates for different vehicle types reflect the different vehicle arrival rates, predefined operating profile, and parking slot allocation?
- RQ4: How do the average numbers of vehicles waiting in a queue for a free parking slot and for exiting the parking lot vary as a function of vehicle arrival rates and the specified operating profile of vehicle types?

These research questions will be addressed through formal modeling, simulation experiments, and performance evaluation to assess the feasibility and benefits of the proposed DSPN-based model of a smart mini parking system.

The remainder of this paper is organized as follows: Section 2 presents an overview of related work on smart parking systems and Petri net-based modeling. Section 3 outlines the research methodology. Section 4 describes the proposed DSPN model, detailing its logic, structure, transitions, and parameters. Section 5 covers the simulation setup and presents the simulation results of the model's performance evaluation. Section 6 discusses the key findings, and Section 7 concludes the paper and highlights directions for future research.

2. RELATED RESEARCH

The development of smart parking systems has gained significant attention in recent years due to increasing urbanization and the need for efficient parking space utilization. Various studies have explored intelligent parking solutions by integrating technologies such as the Internet of Things, machine learning, and Petri net modeling to optimize parking management. This section briefly reviews relevant research on smart parking models, highlighting advancements in those approaches and their application in parking optimization.

In 2008, Yan et al. proposed and evaluated the hardware and software architecture of *SmartParking*, a service-oriented intelligent parking system that enables drivers to seamlessly view and reserve available parking slots in real time, enhancing convenience and reducing search time for optimal parking efficiency [10].

The parking process can be effectively modeled as a birth-death stochastic process, enabling the prediction of revenue trends. These forecasts can then inform strategic business decisions, such as adjusting pricing strategies, introducing promotional discounts, or optimizing parking fees to maximize profitability and enhance customer engagement [11].

Many studies on the modeling of smart parking lots are based on the discrete-event simulation (DES) approach. For instance, the study of Babic et al. (2016) was based on the utilization of DES and a M/M/c/c queueing model to estimate the profitability of electric vehicle chargers at parking lots [12], while Soto-Ferrari et al. (2021) employed DES to evaluate the benefits of smart parking technologies in a Saudi Arabian hospitals using the Arena® simulation software [13]. Discrete-event modeling and the simulation of smart parking conflict management strategies were the focus in [14] and [15]. Attempts to upgrade parking productivity and effectiveness without adding parking slots were made by combining the DES approach with supervised machine learning algorithms to classify parking slots depending on their occupation time so departures could be predicted [16].

When it comes to the use of various classes of Petri nets in the context of smart cities or smart parking solutions, it should be noted that their application remains relatively uncommon in research studies. One of the earliest attempts in this domain is the work of Zhang et al. (2016), which introduces a parking model based on the class of timed Petri nets [17]. This approach was utilized for the modeling and evaluation of key processes, including free parking slot discovery, navigation to available parking slots, and vehicle parking. The model's performance was analyzed through simulations conducted in MATLAB®. Recent works include the case study by Makke and Gusikhin (2020), which focused on developing a cost-efficient smart parking information system for three concrete public parking spaces and the utilization of ordinary (i.e., non-timed) Petri net models in a certain phase of an edge computing process, as an output from CCTV camera images and event logs processing to track real-time parking structure usage [18]. Later, the same authors proposed a design of a robust, IoT-based, adaptive parking information system based on the generation of a corresponding Petri net model, which monitors and records the occupancy status of parking slots [19].

The study of Redjimi (2022), which presents a multi-agent system approach for modeling vehicle environments to efficiently guide drivers to the nearest available parking spaces while navigating through the least congested routes, utilizes reference nets networks, a specific class of object-oriented, high-level Petri nets for which individual tokens can represent whole nets so that a system of nested nets is obtained [20]. His solution also includes the utilization of Reference Nets Workshop (ReNeW), high-level Petri net simulation software that provides a flexible modeling framework for reference nets networks by supporting dynamic net structures and allowing for hierarchical modeling, thus enhancing its applicability in complex system simulations.

Recently, based on the M.Sc. thesis of Souza (2021) [21], Souza and Soares (2023) described a smart city application design approach based on the combined utilization of SysML, an UML-based language for general-purpose modeling in system engineering applications that includes hardware, software, information, processes, people, and procedures, with the class of timed colored Petri nets, a discrete-event modeling and simulation formalism and an extension of colored Petri nets, to tackle the challenges of designing urban traffic signal control systems [22].

Unlike the majority of existing studies, which have focused primarily on large-scale parking infrastructures or overlooked type-specific slot allocation and reservation behavior, this research uniquely models a small-scale, type-aware smart parking system using DSPNs, offering a detailed performance evaluation under varying operational profiles and arrival dynamics.

3. METHODOLOGY

The methodology used to assess the utilization of a smart mini parking system is based on leveraging the class of DSPNs. To the best of the authors' knowledge, this is a pioneering attempt to apply such an approach to the modeling and evaluation of smart parking systems. The class of DSPNs is employed to model a smart mini parking system and analyze its dynamic behavior, capturing both deterministic processes (e.g., reservation policies, billing interval durations) and stochastic events (e.g., vehicle arrivals, parking durations, and departures).

The class of DSPNs is a powerful formalism used to model systems that exhibit both probabilistic (i.e., stochastic) and time-deterministic behaviors. They extend the expressive capabilities of traditional generalized stochastic Petri nets by incorporating deterministic timing alongside exponential and immediate transitions [23].

A hypothetical smart mini parking system, consisting of M = 7 parking slots was modeled and evaluated through numerical simulations (i.e., stationary simulations) of the proposed DSPN-based model using TimeNET® v4.5, a powerful software tool used for the modeling, simulation, and performance analysis of a wide range of Petri nets, including stochastic Petri nets and generalized stochastic Petri nets [24-25]. The stationary simulation/standard mode simulates the steady-state behavior of an arbitrary stochastic Petri net, enabling stationary approximation for concurrent DSPNs in continuous time without the limitation of allowing only one enabled transition with a non-exponentially distributed firing time per marking [26].

Knowing that the operating profile OP refers to the composition, probability distribution, and behavior of different types of vehicles using the smart parking system, this study takes into account four different types of vehicles: (a) handicapped vehicles (i = 1), (b) regular vehicles (i = 2), (c) small utility/cargo vehicles (i = 3), and (d) electric vehicles (i = 4). The seven parking slots are designated for the following four vehicle types: one for handicapped vehicles, three for regular vehicles, one for small utility/cargo vehicles, and two for electric vehicles. The categorization of mini parking slots was done based on anticipated real-world parking needs, the estimated distribution of vehicle types, and the smart mini parking capacity, M. The probability distribution of the four kinds of vehicles is given by the 4-tuple (p_1 p_2 p_3 p_4) defining the mix of vehicles' types in the traffic flow, where p_i (i = 1, 2, 3, 4) are the probabilities of each vehicle type appearance, such that $p_1 + p_2 + p_3 + p_4 = 1$. In this case, the operating profile OP = (0.1 0.4 0.3 0.2) was used.

In TimeNET®, the utilization rate of a single parking slot can be directly obtained by defining the performance measure $\mathrm{Prob}(i)$, where $i \in \{1, 2, ..., M\}$, as specified by Eq. 1. It estimates the steady-state probability that there is a token in places *P_Si_occupied* (i = 1, 2, ..., M) in the DSPN model (Fig. 1), i.e., the stationary probability that the parking slot S_i (i = 1, 2, ..., M) is occupied, given a specific arrival rate of vehicles ($\lambda_{\mathrm{arrival}}$) and a pre-determined operating profile.

$$\mathrm{Prob}(i) = (\#P\_Si\_occupied == 1) \text{ for } i \in \{1, 2, ..., M\} \tag{1}$$

Under these prerequisites, the utilization rate of a smart parking system, comprised of M parking slots, can be calculated using Eq. 2. Note that although the slots are type-specific, Eq. 2 calculates the overall system utilization by aggregating the occupancy probabilities across all seven parking slots.

$$\overline{U} = \frac{1}{M} \cdot \sum_{i=1}^{M} \overline{\mathrm{Prob}(i)} \text{ for } i \in \{1, 2, ..., M\} \tag{2}$$

The variables used in Eq. 2 are as follows:

$\overline{U}$ is the average occupancy rate of the smart parking system;

M is the total number of parking slots;

$i \in \{1, 2, ..., M\}$ is an index of each parking slot;

$\overline{\mathrm{Prob}(i)}$ is the average steady-state probability that the parking slot $i \in \{1, 2, ..., M\}$ is occupied, based on multiple simulation runs.

Box and whisker plots were generated for each parking slot to analyze the variability and distribution of utilization rates across different parking slots under repeated simulation runs for a fixed arrival rate $\lambda_{\mathrm{arrival}}$ and a pre-determined operating profile. Box and whisker plots are widely used in simulation-based performance evaluation to capture the variability across multiple runs and highlight central trends and dispersion [27].

Given a specific arrival rate of vehicles ($\lambda_{\mathrm{arrival}}$) and a pre-determined operating profile, and given that parking slot reservations are very short compared to parking slot occupations, the averaged reservation rates per parking slot $\overline{R_i}$, $i \in \{1, 2, ..., M\}$ can be roughly estimated by Eq. 3.

$$\overline{R_i} = \frac{ENR(i)}{ERD(i)} \tag{3}$$

The variables used in Eq. 3 are as follows:

$\overline{R_i}$ is the average reservation rate per parking slot $i \in \{1, 2, ..., M\}$ (i.e., the rate of how many reservations occur per unit of time);

$ENR(i)$ is the expected number of reservations made for the parking slot $i \in \{1, 2, ..., M\}$;

$ERD(i)$ is the expected reservation duration for the parking slot $i \in \{1, 2, ..., M\}$;

In the DSPN model shown in Fig. 1, $ENR(i)$ and $ERD(i)$ for the parking slot $i \in \{1, 2, ..., M\}$ are represented by the expected number of tokens in the place *P_Si_reserved*, $i \in \{1, 2, ..., M\}$ and the average time delay $1/\pi_{\mathrm{park\_to\_Si}}$, respectively.

The expected number of tokens in the place *P_Si_reserved*, $i \in \{1, 2, ..., M\}$ indicates how many tokens are present in that place on average at any given steady-state moment. It reflects the steady-state probability-weighted average number of reservations, $ENR(i)$, $i \in \{1, 2, ..., M\}$, currently ongoing in each slot at any random time and can be estimated directly through simulation using the measure defined by Eq. 4.

$$ENR(i) = (\#P\_Si\_reserved) \text{ for } i \in \{1, 2, ..., M\} \tag{4}$$

The time delay $1/\pi_{\text{park\_to\_Si}}$ is the average firing delay of the exponential transition *T_park_to_Si*, $i \in \{1, 2, ..., M\}$. It represents the average time elapsed from the moment a new reservation is made until a vehicle is parked in its assigned slot (i.e., the average duration for which a single reservation remains active). In this particular case, $1/\pi_{\text{park\_to\_Si}}$, $i \in \{1, 2, ..., M\}$, is set to 10 minutes.

The number of vehicles waiting in a queue for a free parking slot, V_{arrival}, is represented by the expected number of tokens residing in place *P_vehicles_arrival* in the DSPN model (Fig. 1) and can be estimated with the TimeNET® measure defined by Eq. 5, while the number of vehicles waiting in a queue to leave the mini parking, $V_{\text{departure}}$, is represented by the expected number of tokens residing in place *P_vehicles_departure* (Eq. 6).

$$V_{\text{arrival}} = (\#P\_vehicles\_arrival) \tag{5}$$

$$V_{\text{departure}} = (\#P\_vehicles\_departure) \tag{6}$$

4. THE PROPOSED MODEL

Two excerpts from the proposed DSPN-based model of a smart mini parking system, which refer to handicapped and electric vehicles, are portrayed in Figs. 1a and 1b, respectively. The same names of places, transitions, and parameters that appear in both excerpts refer to single elements in the overall DSPN model. While modeling, it was assumed that (a) all time-consuming processes in the system (e.g., vehicles' arrivals and departures, reservation durations, and parking durations), except the pre-determined (i.e., deterministic) times, are exponentially distributed and (b) for simplicity, once a reservation of a parking slot is made, it cannot be cancelled. Below is a brief explanation of the logic/behavior captured by the two model excerpts, without delving into the specifics or meanings of the DSPN building blocks used.

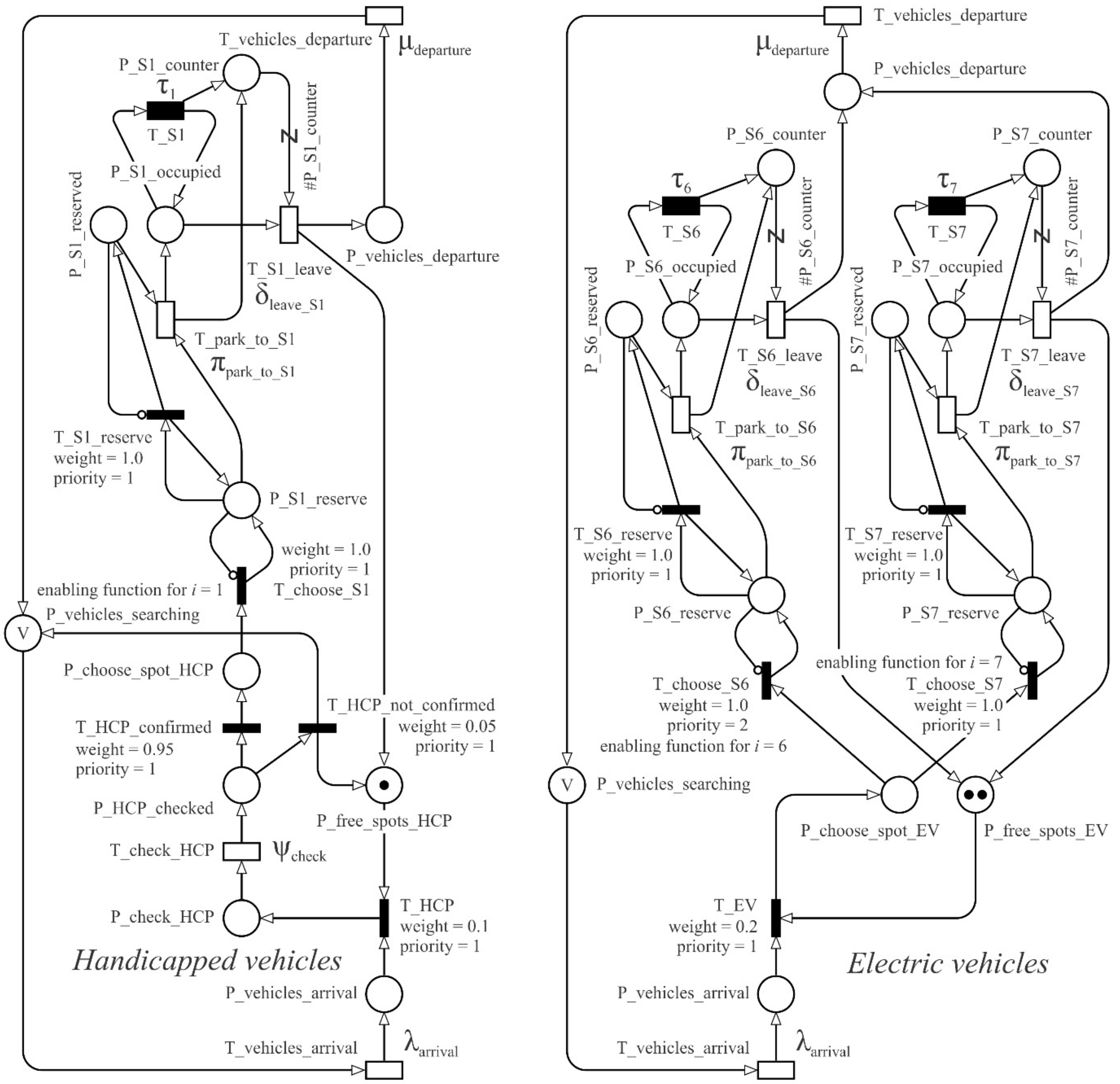
Initially, the place *P_vehicles_searching* contains *V* tokens, representing the total number of vehicles in the system searching for a free parking slot, arriving at a rate of λ_{arrival}. The firing of immediate transitions *T_HCP* (weight = 0.1) (Fig. 1a) and *T_EV* (weight = 0.2) (Fig. 1b) is decided for each vehicle type according to the predefined operating profile *OP* = (0.1 0.4 0.3 0.2), describing the probability distribution of handicapped, regular, utility/cargo, and electric vehicles, respectively. The weight value is a real number (default: 1.0), which specifies the relative firing probability of the immediate transition concerning other simultaneously enabled immediate transitions that are in conflict. These transitions are enabled and fire only if there is at least one token in places *P_free_spots_HCP* and *P_free_spots_EV*, respectively (i.e., if there is a free parking slot of the corresponding vehicle type). The current number of tokens in each of these places in the DSPN model represents the current number of free parking slots.

Especially for handicapped vehicles (Fig. 1a), a mechanism for checking their validity is modeled through a process that lasts, on average, $1/\psi_{\text{check}}$ minutes and starts when a token is placed in the place *P_check_HCP*. It is assumed that 95% of vehicles that claim to belong to handicapped persons are handicapped vehicles, which is represented by firing of the immediate transition *T_HCP_confirmed* (weight = 0.95), while the remaining 5% of vehicles are rejected by firing of the immediate transition *T_HCP_not_confirmed* (weight = 0.05).

The reservation process starts when a token is placed in places *P_choose_spot_HCP* (Fig. 1a) and *P_choose_spot_EV* (Fig. 1b) for handicapped and electric vehicles, respectively.

Especially in cases where there is more than one parking slot for a vehicle type, such as in the case of electric vehicles (Fig. 1b), the reservation is made according to the priority value assigned to immediate transitions *T_choose_Si* (i = 2, 3, 4, 6, 7). The default priority is 1, with higher values indicating higher firing priorities. The priority value is a natural number that defines the precedence among simultaneously enabled immediate transitions to prevent conflicts among them. In addition, the enabling functions for immediate transitions *T_choose_Si* (i = 1, 2, …, 7), which are defined as

(#*P_Si_reserved* == 0) AND (#*P_Si_occupied* == 0) for $i \in \{1, 2, ..., M\}$, assure that the reservation of a specific parking slot can be made only if it is not already reserved or occupied.



(a) Handicapped vehicles (1 slot) (b) Electric vehicles (2 slots)

Fig. 1. Excerpts from the overall DSPN-based model of a smart mini parking system (Source: the authors)

When a parking slot is chosen, represented by a token in the place *P_S1_reserve* (Fig. 1a) or in one of the places *P_S6_reserve* or *P_S7_reserve* (Fig. 1b), the reservation becomes active by placing a token in the place *P_S1_reserved* (Fig. 1a) or in one of the places *P_S6_reserved* or *P_S7_reserved* (Fig. 1b). It lasts, on average, for $1/\pi_{\text{park\_to\_Si}}$ minutes (i = 1, 2, …, 7). While the reservation is active, the vehicle arrives at the mini parking lot and occupies the pre-reserved parking slot, which is represented by removing the token from place *P_S1_reserve* (Fig. 1a) or places *P_S6_reserve* or *P_S7_reserve* (Fig. 1b), which represents the ending of a slot reservation, and by placing a token in the place *P_S1_occupied* (Fig. 1a) or in one of the places *P_S6_occupied* or *P_S7_occupied* (Fig. 1b), which represents the beginning of an active parking session. This automatically puts a token in the place *P_Si_counter* (i = 1, 2, …, 7), which starts a new billing interval for charging parked vehicles, and enables the deterministic transition *T_S1* (Fig. 1a) or one of the deterministic transitions *T_S6* or

T_S7 (Fig. 1b), which measure the pre-determined times τ_i h (i = 1, 2, …, 7) of billing interval durations. Meanwhile, the exponential transitions *T_Si_leave* (i = 1, 2, …, 7) are also enabled and fire in $1/\delta_{\text{leave\_Si}}$ minutes (i = 1, 2, …, 7), on average, meaning that the vehicle has ended its parking session, has left the parking slot, and has departed (a token in the place *P_vehicle_departure*). If the vehicle still occupies the parking slot when deterministic transitions *T_Si* (i = 1, 2, …, 7) fire, the parking session continues by placing an additional token in places *P_Si_counter* (i = 1, 2, …, 7).

Once a vehicle vacates the parking slot, represented by a token placed in the place *P_vehicles_departure*, it exits the mini parking area after an average time of $1/\mu_{\text{departure}}$ minutes.

5. SIMULATION

5.1. Setup

A series of numerical simulations of the DSPN model have been carried out in TimeNET® for a pre-determined set of vehicles' arrival rates $\lambda_{\text{arrival}} = 1/\overline{T}$, where $\overline{T} \in [0.50; 10.00]$ and a step of 0.50 is the mean inter-arrival time of vehicles, expressed in minutes. This means that a vehicle looking for a free parking lot arrives in the system every x minutes on average ($1/\lambda_{\text{arrival}} = x$ minutes). These values have been converted into hours. Each simulation run was conducted assuming the setup parameters shown in Table 1.

Table 1

Parameter values for steady-state continuous time simulation (Source: the authors)

Parameter	Setting
Detect initial transient	ON
Confidence level [%]	95%
Max. relative error [%]	1%
Permitted difference for probability measures close to 0.0 or 1.0 [%]	1%
Seed value	12,345
Min. # of firings for each transition	5,000
Max. real time [sec]	86,400
Method for statistical analysis	Spectral variance

For each $\overline{T} \in [0.50; 10.00]$ minutes, a series of 15 simulation runs was initiated to acquire more accurate estimated averaged values for all measures of interest. The working parameters used in the DSPN model are listed in Table 2. Since the simulation is carried out under stationary (i.e., steady-state) assumptions, the results obtained based on these working parameters represent long-run average behaviors rather than specific times of day or weekdays, and they correspond to the operating profile specified previously, *OP* = (0.1 0.4 0.3 0.2), and the arrival rate of vehicles, λ_{arrival}. However, since each period, or even an entire weekday in real-world conditions, can be characterized by a specific operating profile and arrival rate, the same DSPN model can be utilized to evaluate the performance of the smart mini parking system under that particular profile and arrival rate.

Rows 5 and 6 in Table 2 show that different type-specific parking durations (i.e., park slot occupancy times) are taken into account and modeled using different mean exponential firing delays $1/\delta_{\text{leave\_Si}}$ of the exponential transitions *T_Si_leave* (i = 1, …, 7) in the DSPN model, equal to 2.50 hours for all vehicle types, except for utility/cargo vehicles (i = 1, 2, 3, 4, 6, 7), and 15.00 minutes = 0.25 hours solely for utility/cargo vehicles (i = 5). Similarly, rows 7 and 8 in Table 2 refer to different type-specific billing intervals (i.e., the fixed time durations used for charging parked vehicles). These are defined by the fixed firing time τ_i of the deterministic transition *T_Si* (i = 1, …, 7) in the DSPN model, equal to 1.00 hour for all vehicle types, except for utility/cargo vehicles (i = 1, 2, 3, 4, 6, 7), and 20.00 minutes = 0.33 hours solely for utility/cargo vehicles (i = 5).

Table 2

Working parameters for the DSPN model (Source: the authors)

	Parameter	Designation	Value(s)	Unit
1	Total number of vehicles in the system	V	10	///
2	Mean inter-arrival time of vehicles entering the smart mini parking system	$1/\lambda_{\text{arrival}}$	[0.50; 10.00] step 0.50	minute
3	Mean inter-departure time of vehicles exiting the smart mini parking system	$1/\mu_{\text{departure}}$	5.00	minute
4	Average reservation time (time to park)	$1/\pi_{\text{park\_to\_Si}}$ (i = 1, …, 7)	10.00	minute
5	Average park slot occupancy time for all vehicle types, except for utility/cargo vehicles	$1/\delta_{\text{leave\_Si}}$ (i = 1, 2, 3, 4, 6, 7)	2.50	hour
6	Average park slot occupancy time for utility/cargo vehicles	$1/\delta_{\text{leave\_Si}}$ (i = 5)	15.00	minute
7	Billing interval for all vehicle types, except for utility/cargo vehicles	τ_i (i = 1, 2, 3, 4, 6, 7)	1.00	hour
8	Billing interval for utility/cargo vehicles	τ_i (i = 5)	20.00	minute
9	Average check duration for handicapped vehicles	$1/\psi_{\text{check}}$	5.00	minute

5.2. Results

Fig. 2 represents a visualization of the box and whisker plots of parking slot utilization rates for mean inter-arrival times of $1/\lambda_{\text{arrival}}$ = 1 minute (upper plots) and $1/\lambda_{\text{arrival}}$ = 10 minutes (lower plots) based on 15 simulation runs for different vehicle types.

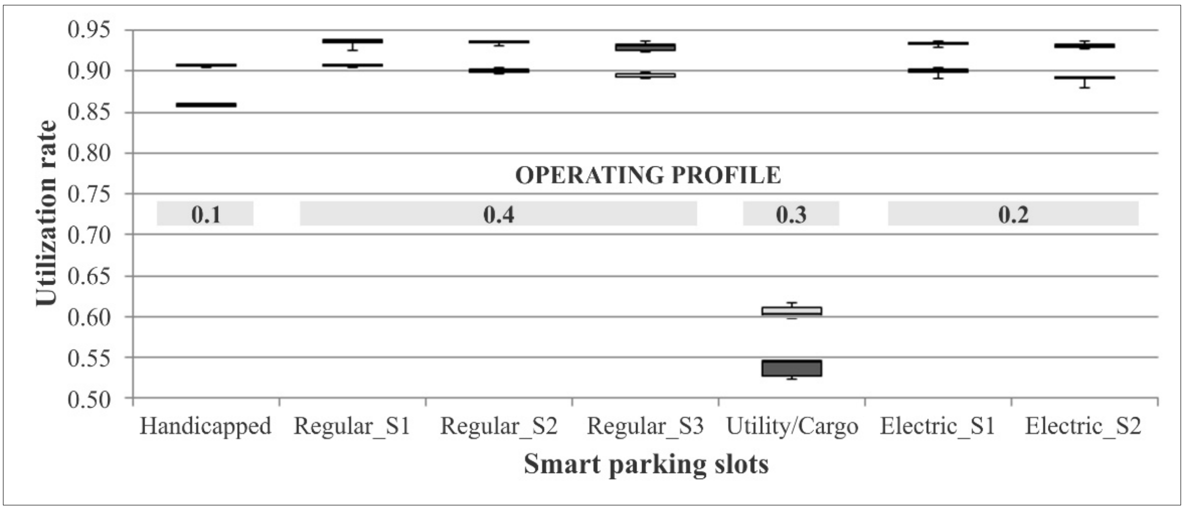


Fig. 2. Box and whisker plots of smart parking slot utilization rates by vehicle types (Source: the authors)

Fig. 3 shows the average utilization rate of the entire smart mini parking system, calculated using Eq. 2, as a function of vehicles' mean inter-arrival times ($1/\lambda_{\text{arrival}}$), along with the corresponding trend line.

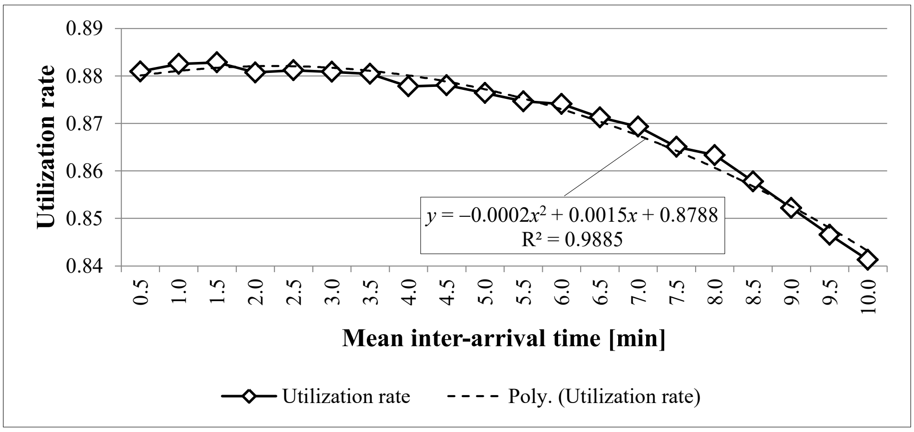


Fig. 3. Average utilization rate as a function of mean inter-arrival times (Source: the authors)

The stacked column chart in Fig. 4 portrays the aggregated average reservation rates for each vehicle type's parking slot per unit of time (one hour), computed according to Eq. 3.

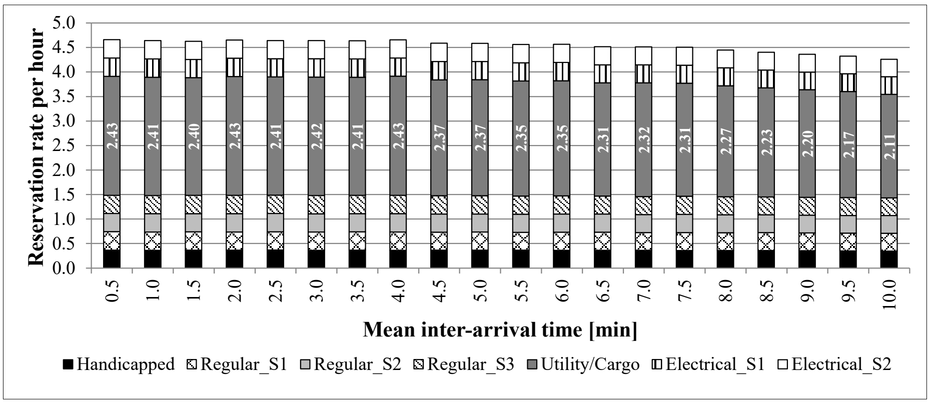


Fig. 4. Aggregated reservation rates per hour, as a function of mean inter-arrival times (Source: the authors)

Average reservation rates per unit of time (one hour) for three parking slots are shown in Fig. 5.

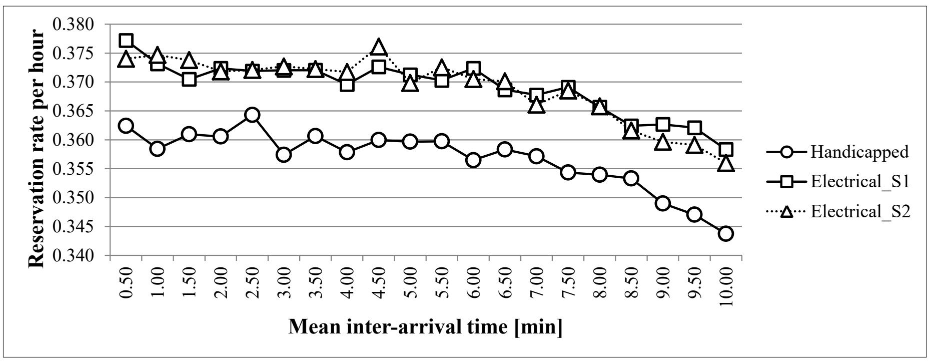


Fig. 5. Average reservation rates per hour for handicapped vehicles (one slot) and electric vehicles (two slots) as a function of vehicles' mean inter-arrival times (Source: the authors)

Fig. 6 shows the average number of vehicles waiting in a queue for a free parking slot and for exiting the smart parking lot, presented as a function of mean inter-arrival times ($1/\lambda_{\text{arrival}}$), along with the corresponding trend lines.

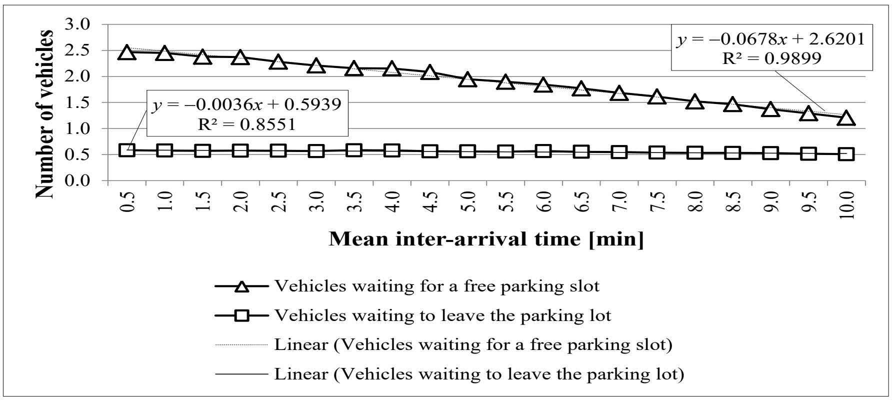


Fig. 6. Average number of vehicles waiting for a parking slot and for leaving the parking lot as a function of vehicles' mean inter-arrival times (Source: the authors)

6. DISCUSSION

This section discusses and interprets the simulation results obtained from the DSPN-based model evaluation of the proposed hypothetical smart mini parking system.

The verification of the DSPN model was performed using the TimeNET®'s *Tokengame* module, which enables interactive simulation of model behavior without considering transition firing times. This module was used to debug, inspect, and confirm that the DSPN model operates as intended.

While model validation is essential in most modeling studies, it was not feasible here due to several reasons: (a) a lack of access to real-world data from existing smart mini parking systems, preventing calibration or empirical validation; (b) the study's focus is on understanding dynamics, testing assumptions, and scenario comparison rather than replicating a real system; (c) the fact that the modeled system is hypothetical and not yet deployed, leaving no benchmark for validation; and (d) the use of well-defined simulation scenarios with controlled inputs in evaluations, ensuring internal consistency and responsiveness.

Box and whisker plots shown in Fig. 2, generated for mean inter-arrival times $1/\lambda_{\text{arrival}} \in \{0.50, 10.00\}$ minutes, indicate that simulation results are highly consistent, with small interquartile ranges, suggesting stable system behavior. The narrow spread implies that utilization rates are predictable and close to the median, aiding reliable planning and forecasting. Tight distributions reflect the DSPN model's robustness, especially for parking slots with well-matched demand and supply. Moreover, Fig. 2 shows that the small utility/cargo vehicles' parking slot is underutilized compared to the other parking slots.

A key insight from Fig. 2 is the discrepancy between the expected vehicle mix from the operating profile OP = (0.1 0.4 0.3 0.2) and the actual parking slot utilization rates. For instance, handicapped vehicles (10% arrival probability) show higher utilization rate than utility/cargo vehicles (30% arrival probability), even though both have only one dedicated slot. This likely results from mismatched demand and supply: handicapped vehicles may frequently find their slot occupied due to longer parking durations, while utility/cargo vehicles have shorter stays, resulting in a lower average utilization rate. Thus, the utilization rate depends not just on arrival probabilities but also on how well slot supply matches demand and the duration of stay. Even with lower arrival rates, longer occupancy times can increase the utilization rate, as seen with handicapped vehicles.

According to Fig. 3, the average utilization rate for the whole smart mini parking system continually decreases non-linearly as the vehicles' arrival rate λ_{arrival} decreases, which is in line with real-world expectations. The curve of this decreasing trend can be well fitted with a polynomial regression trendline of order 2 using the least squares method, which yields a high coefficient of determination (R-squared = 0.9885). This means that the fitted curve explains 98.85% of the variability in the underlying data, confirming that the chosen fitting model captures the underlying relationship between vehicle arrival rates and parking slot utilization with high accuracy. Since a low-order polynomial is used, there is no risk of overfitting.

The findings depicted in Fig. 4 suggest that the average number of reservations per hour for handicapped, regular, and electric vehicles remains relatively low and stable, ranging between approximately 0.34 and 0.38 across varying vehicle mean inter-arrival times. In contrast, small utility/cargo vehicles exhibit substantially higher reservation intensity, with values ranging between 2.11 and 2.43 reservations per hour. This discrepancy suggests a disproportionate demand for the slot for utility vehicles, potentially due to an undersupply of such slots (one slot) relative to their share in the operating profile (30%) or to the shorter average occupancy time ($1/\delta_{\mathrm{leave\_S5}}$ = 15 minutes), which increases turnover pressure. The relatively flat behavior of reservation rates for the other categories also implies that their designated slots are sufficient under current load assumptions.

Still, Fig. 5 indicates that, as expected, the average reservation rates per hour decrease non-linearly as the mean inter-arrival time increases, not only for handicapped and electric vehicles but also for other vehicle types (not shown for simplicity). However, a small plateau may appear for certain vehicle types (e.g., those with low demand or abundant available slots), for which the reservation rate remains relatively constant under low traffic conditions simply because demand stays well below capacity. Nevertheless, as the inter-arrival time continues to rise, reservation rates eventually decline. The lower reservation rates for handicapped vehicles compared to electric vehicles are due to their smaller share in the operational profile (10% vs. 20%).

As expected, Fig. 6 shows that the average number of vehicles waiting for a free parking slot decreases steadily in a linear fashion as the arrival rate $\lambda_{\mathrm{arrival}}$ decreases, while the number of vehicles leaving the parking lot barely decreases linearly despite the fixed value of the departure rate $\mu_{\mathrm{departure}}$, mainly due to the decreasing number of vehicles in the system as a result of the decreasing arrival rate.

7. CONCLUSIONS

This study presented a detailed modeling and performance evaluation of a smart mini parking system using DSPNs. The proposed model incorporated essential features such as vehicle-type-specific parking slots, time-based arrivals and departures, and reservation mechanisms to reflect realistic operational behavior. A simulation-based analysis was conducted across varying arrival delays and a predefined vehicle-type distribution to examine the system's responsiveness and efficiency from multiple perspectives, including parking slot utilization rates, average reservation rates, and vehicle waiting queue lengths while arriving and departing. The findings provide valuable insights into the dynamics of smart parking environments and highlight critical factors that influence system performance and resource allocation, confirming the significant potential of smart mini parking systems in addressing critical urban mobility challenges.

Smart mini parking systems address urban mobility challenges by optimizing limited space through real-time reservations, type-specific slots, and digital management. They reduce congestion and idling, support electric vehicles, ensure accessibility, and promote sustainable, scalable solutions for smarter, more efficient cities.

DSPN modeling offers multiple benefits for smart parking, including (a) hybrid time handling (stochastic and deterministic); (b) accurate representation of real-world behavior; (c) effective modeling of concurrency and parallelism (e.g., simultaneous arrivals, reservations, and occupations); (d) resource limitation via tokens; (e) support for complex logic like reservations and timeouts; and (f) the ability to compute key performance metrics such as utilization rates, waiting times, throughput, and reservation success rates.

Despite its many benefits, DSPN modeling exhibits several limitations: (a) the model can become highly complex and difficult to manage, especially as the number of vehicle types, parking slots, and rules increases; (b) DSPNs may not scale well for large-scale systems (e.g., city-wide parking systems), as the state-space and computation time grows rapidly, possibly leading to computationally intractable models; (c) simulation results may be sensitive to parameter settings (e.g., arrival rates, parking durations), requiring careful calibration and validation; (d) simplified behaviors (e.g., idealized reservations, no cancellations, constant routing) may omit real-world uncertainties like human errors or system failures; and (e) specialized tools (e.g., TimeNET®) and technical expertise are required to build, verify, evaluate, validate, and interpret the obtained results correctly.

While the model lacks calibration due to the absence of empirical data, it remains effective for exploring dynamics, comparing scenarios, and evaluating the influence of key parameters under hypothetical but realistic operating conditions. However, its effectiveness is limited to relative performance evaluation and behavioral analysis under controlled conditions.

Future work could expand upon the current work in several ways. It may integrate real-time data from an operational smart parking system to enable full model calibration and empirical validation after the existing DSPN model is adjusted/updated to capture the specifics of that real-world system. This would enhance the model's realism and applicability. Additionally, exploring dynamic pricing strategies, user behavior modeling (e.g., cancellations or no-shows), and adaptive allocation mechanisms under varying demand patterns would provide deeper operational insights. The scalability of the model could also be investigated by extending the framework to somewhat larger parking facilities. Finally, comparative studies with alternative modeling techniques, such as agent-based simulation, may reveal strengths and trade-offs in terms of accuracy, complexity, and computational efficiency.

References

1. Abdi, H. & Shahbazitabar, M. Smart city: A review on concepts, definitions, standards, experiments, and challenges. *Journal of Energy Management and Technology (JEMT)*. 2020. Vol. 4. No. 3. P. 1-6. DOI: 10.22109/jemt.2020.206444.1205.
2. Talebkhah, M. & Sali, A. & Gordan, M. et al. Comprehensive review on development of smart cities using industry 4.0 technologies. *IEEE Access*. 2023. Vol. 11. P. 91981-92030. DOI: 10.1109/ACCESS.2023.3302262.
3. *Annual car sales worldwide 2010-2024, with a forecast for 2025*. 2025. Available at: https://www.statista.com/statistics/200002/international-car-sales-since-1990/.
4. *International automobile sales by region 2018-2024*. Available at: https://www.statista.com/statistics/200005/international-car-sales-by-region-since-1990/.
5. Paulus, N. *Car Ownership Statistics in the US*. 2025. Available at: https://www.moneygeek.com/resources/car-ownership-statistics/.
6. Altendorf, N. *Rekord! So viele Autos wie noch nie in Deutschland*. 2024. Available at: https://www.bild.de/news/inland/trotz-verkehrswende-rekordhoch-49-1-millionen-pkw-in-deutschland-6704f31ef31f1f357a0051a3.
7. *U.S. Cities with the Highest and Lowest Vehicle Ownership*. 2025. Available at: https://www.titlemax.com/discovery-center/u-s-cities-with-the-highest-and-lowest-vehicle-ownership/.
8. *Urban road traffic congestion: Traffic jams in cities*. Union of International Associations (UIA): Encyclopedia of World Problems and Human Potential. 2020. Available at: https://encyclopedia.uia.org/problem/urban-road-traffic-congestion.
9. Gholipour, H.F. & Esfandiar, K. & Rasoolimanesh, M. *Visitor caps: what potential impact on the world tourism industry?* Economic Observatory: Transport & Infrastructure. 2024. Available at: https://www.economicsobservatory.com/visitor-caps-what-potential-impact-on-the-world-tourism-industry?
10. Yan, G. & Olariu, S. & Weigle, M.C. & Abuelela, M. SmartParking: A secure and intelligent parking system using NOTICE. In: *Proceedings of the 2008 11th International IEEE Conference on Intelligent Transportation Systems*. Beijing. 2008. P. 569-574. DOI: 10.1109/ITSC.2008.4732702.
11. Yan, G. & Yang, W. & Rawat, D.B. & Olariu, S. SmartParking: A secure and intelligent parking system. *IEEE Intelligent Transportation Systems Magazine*. 2011. Vol. 3. No. 1. P. 18-30. DOI: 10.1109/MITS.2011.940473.

12. Babic, J. & Goncalves de Carvalho, A. & Ketter, W. & Podobnik, V. Modeling smart parking lots through a discrete-event simulation. In: *Winter Conference on Business Intelligence (WCBI-16)*. Snowbird, UT. USA, 2016.
13. Soto-Ferrari, M. & Romero-Rodriguez, D. & Escorcia-Caballero, J.P. et al. A discrete-event simulation approach to assess the benefits of parking technologies in hospitals. *International Journal of Operations and Quantitative Management*. 2021. Vol. 27. No. 2. P. 141-152. DOI: 10.46970/2021.27.2.4.
14. Dominici, A. & Capocchi, L. & De Gentili, E. & Santucci, J.-F. Discrete event simulation of smart parking conflict management. In: Kim, S. & Feng, B. & Smith, K. et al. (eds.) In: *Proceedings of the 2021 Winter Simulation Conference (WSC '21)*. Phoenix, AZ, USA, 2021.
15. Dominici, A. & Capocchi, L. & De Gentili, E. & Santucci, J.-F. Discrete event modeling and simulation of smart parking conflict management. In: *Proceedings of the 24th International Congress on Modelling and Simulation.* Sydney, Australia, 2021. P. 246-252.
16. Dominici, A. & Capocchi, L. & De Gentili, E. & Santucci, J.-F. Towards a combination of discrete-event simulation with machine learning for smart-parking. In: *Proceedings of the 2020 Summer Simulation Conference (SummerSim '20)*. Virtual Event. Spain, 2020.
17. Zhang, X. & Li, D. & Wang, J. et al. Faster parking and less cruise for public parking spot discovery: modeling and analysis based on timed Petri nets. In: *Proceedings of 2016 IEEE 13th International Conference on Networking, Sensing, and Control (ICNSC 2016)*. Mexico City, Mexico, 2016. DOI: 10.1109/ICNSC.2016.7478970.
18. Makke, O. & Gusikhin, O. Petri net-based smart parking information system. In: *Proceedings of the 6th International Conference on Vehicle Technology and Intelligent Transport Systems (VEHITS 2020).* Prague, Czech Republic. 2020. P. 385-393.
19. Makke, O. & Gusikhin, O. Robust IoT based parking information system. In: Klein, C. & Helfert, M. & Berns, K. & Gusikhin, O. (eds.) *Smart Cities, Green Technologies, and Intelligent Transport Systems*. SMARTGREENS 2020, VEHITS 2020. Communications in Computer and Information Science (CCIS). Springer, Cham, Switzerland. 2021. Vol. 1475. P. 204-227.
20. Redjimi, K. Modelling and simulation for smart parking. *Journal of Software Engineering and Simulation*. 2022. Vol. 8. No. 1. P. 25-30.
21. Souza, L.S. *A SysML Profile for Smart City Applications*. MSc thesis. Universidade Federal de Sergipe, Centro de Ciências Exatas e Tecnologia, São Cristóvão – Sergipe, Brasil. 2021. 91 p.
22. Souza, L.S. & Soares, M.S. Combining SysML and timed coloured Petri nets for designing smart city applications. *Journal of Universal Computer Science*. 2023. Vol. 29. No. 10. P. 1217-1249. DOI: https://doi.org/10.3897/jucs.97170.
23. Ciardo, G. & Lindemann, C. Analysis of deterministic and stochastic Petri nets. In: *Proceedings of the 5th International Workshop on Petri Nets and Performance Models*. Toulouse, France' 1993. P. 160-169. DOI: 10.1109/PNPM.1993.393454.
24. *TimeNET: The TimeNET tool project*. 2018. Available at: https://timenet.tu-ilmenau.de/.
25. Zimmermann, A. Modelling and Performance Evaluation with TimeNET 4.4. In: Bertrand, N. & Bortolussi, L. (eds.) *Quantitative Evaluation of Systems.* QEST 2017. Lecture Notes in Computer Science (LNCS). Springer, Cham, Switzerland. DOI: 10.1007/978-3-319-66335-7_19. 2017. Vol. 10503. P. 300-303.
26. Zimmermann, A. & Knoke, M. *TimeNET 4.0 User Manual: A Software Tool for the Performability Evaluation with Stochastic and Colored Petri Nets*. Berlin: Faculty of EE&CS. Technical Report 2007-13. Real-Time Systems and Robotics Group, Technische Universität Berlin. Berlin, Germany. 2007.
27. Law, A.M. *Simulation Modeling and Analysis*. Fifth Edition. New York: McGraw-Hill Education. 2015. 325 p.

Received 03.07.2024; accepted in revised form 18.08.2025