Volodymyr SAKHNO, Viktor POLIAKOV, Oleksiy TIMKOV
National Transport University
Suvorova 1, 01010, Kyiv, Ukraine
Oleksandr KRAVCHENKO*
Zhytomyr State Technological University
Chernykhovsky 103, 10005, Zhytomyr, Ukraine
**Corresponding author*. E-mail: avtoapg@gmail.com

LORRY CONVOY STABILITY TAKING INTO ACCOUNT THE SKEW OF SEMITRAILER AXES

Summary. The mathematical model of a saddle-type lorry convoy is improved taking into account the skew of the axes of a semitrailer. It is shown that the motion of a lorry convoy without the skew of the axes is asymptotically steady. The skew of any axis of a semitrailer results in the worsening of stability of the rectilinear motion conditioned by the oscillations of the towed link.

УСТОЙЧИВОСТЬ АВТОПОЕЗДА С УЧЕТОМ ПЕРЕКОСА ОСЕЙ ПОЛУПРИЦЕПА

Аннотация. Усовершенствована математическая модель седельного автопоезда с учетом перекоса осей полуприцепа. Показано, что движение автопоезда без перекоса осей является асимптотически устойчивым. Перекос любой оси полуприцепа приводит к ухудшению устойчивости прямолинейного движения, обусловленного колебаниями прицепного звена.

1. INTRODUCTION

It is a well-proven long-term practice of exploitation of vehicles that lorry convoys have substantial advantages over single automobiles. Among the most essential operating properties of lorry convoys that provide their safety of motion, it is necessary to mention the motion road holding, as the loss of the motion road holding of the vehicle mostly results in a traffic accident.

The parameters of the motion road holding of a lorry convoy are closely related to the construction of its undercarriage. The practice of exploitation of lorry convoys testifies that even new trailers and semitrailers do not have the skew of the axes conditioned by the perpendicularity of their alignment relative to the longitudinal axis of a semitrailer [1 – 5]. The fact of the presence of such skew of the axes is confirmed by the long-run observations in the process of exploitation of lorry convoys [6].

It is obvious that even at the identical technical state of the undercarriage of a truck tractor and a semitrailer at their making, after some period of exploitation, it is possible to find the different degree of wear of tires and of axle suspension elements of the truck-tractor and the semitrailer. It is known that the axes' alignment angles, loading on wheels, lateral forces, tangential forces (pulling and braking) and pressure of air on tires have an influence on the intensity of their wear. In a quantitative relation these factors are not identical for each of the axes of a lorry convoy [7 – 9]. Consequently, at

the different axes alignment angles and different treadwear, it is possible to speak about the change of the coefficients of resistance to the lateral skid of the wheels and, as a result, the change of the parameters of turnability and stability of a lorry convoy on the whole.

The solving of this problem is a complex task, requiring a study of the processes that take place in the undercarriage of a lorry convoy, both in the process of its making and under the influence of the operating factors.

The aim of the work is a search for the estimation methods of the lorry convoy motion stability, taking into account the angles of alignment of the axes of lorry convoy links.

The theoretical aspects of the analysis of motion stability are based on the mathematical models of the rectilinearly guided motion of a lorry convoy, taking into account the axis alignment angles. Differential equations of the perturbed motion of a lorry convoy are acquired on the basis of these mathematical models. The solution of the equations carried out by the computer program is allowed to forecast the behaviour of a lorry convoy in transient and stationary traffic conditions.

2. PREVIOUS RESEARCH

The theory of motion stability of the wheeled cars is based on the mathematical apparatus of investigating the differential equations developed by A.M. Liapunov [10]. He determined the properties of the perturbed system state consisting in a tendency to the renewal of the trajectories of the unperturbed motion that took place before the appearance of perturbance. The time of return of motion parameters to the initial ones can be accepted for the quantitative estimation of this property. At an oscillation process of the return of these parameters to the initial ones, it is possible to quantitatively estimate the stability according to the damping decrement, i.e. by the degree of reduction of vibration amplitude.

In the works of Lobas L.G. [11, 12], the problems of mechanics of the multilink systems with rolling are considered. The author worked out more perfect mathematical models of the systems with rolling. The new problem statements are given. The methods of their investigations are developed. The influence of design factors on the motion of the constrained systems with classic nonholonomic constraints, generated by the perfectly rigid rolling bodies and with nonholonomic constraints, arising at the rolling of elastic solids is analysed.

For the holding of a lorry convoy on the planned trajectory during the oscillating transitional process, a driver must turn a steering wheel alternately to both sides. In this case, to drive a lorry convoy is more difficult than at a noncyclic transitional process. Furthermore, if the period of an oscillating process is approximately the same as the time of reaction of the system, "an automobile—a driver", then the actions of the driver can be the reason of the continuous oscillating process. The period of oscillations must exceed the time of the reaction of the system under consideration by no less than 3 to 4 times, i.e. it must take more than 4 to 6 sec.

Steady motion, according to Liapunov, is realised in the preliminary unknown area of the initial perturbances, which is called the area of attraction of unperturbed motion. There is a task of determining the borders of this area.

3. RESULTS OF THE RESEARCH

The mathematical model of the plane-parallel motion of a lorry convoy, taking into account the skew of the truck-tractor axles and the semitrailer axes, is developed for the estimation of the stability of the motion of a lorry convoy. As an object of modelling, a double-link lorry convoy consisting of a biaxial truck-tractor and a triaxial semitrailer with connivent uncontrolled axes and having a skew of the axles and axes, as the most typical according to a layout drawing among widespread lorry convoys, is chosen.

Let us consider a lorry convoy as a system that consists of two separate modules—a truck-tractor with a mass centre at the point C_1 and the framework of a semitrailer with a mass centre at the point

C_2 (Fig. 1). A truck-tractor has steerable wheels with the rotation angles Θ_1 and Θ_{1r}, where $\Theta_1 \geq \Theta_{1r}$, and movable details of a steering gear and a steering linkage.
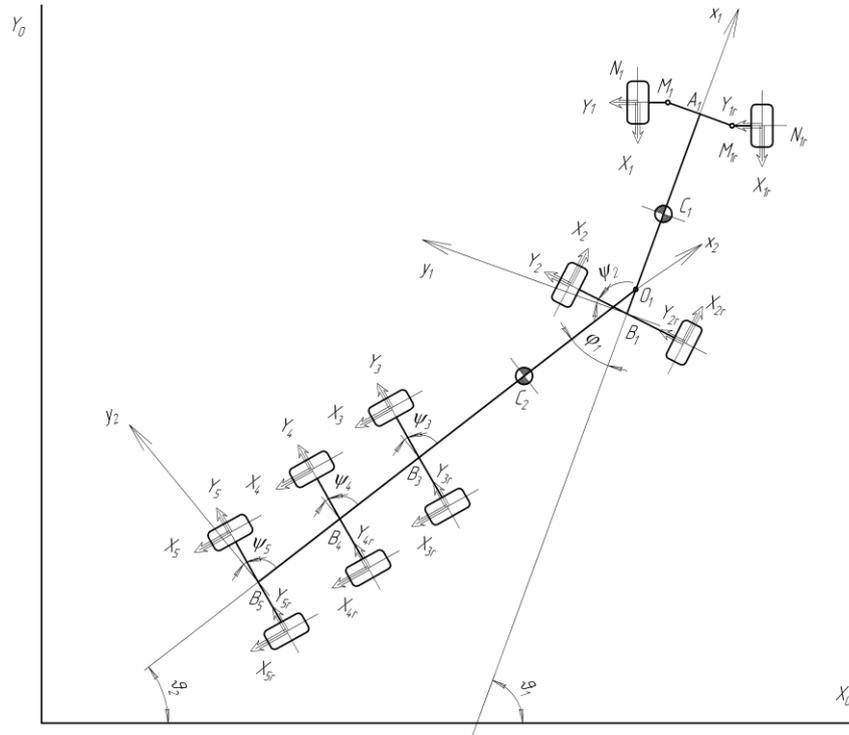
Fig. 1. Design diagram of a double-link lorry convoy at the unsteady turn
Рис. 1. Расчетная схема двухзвенного автопоезда на неустановившемся повороте

The following symbols are accepted in the diagram and further in the text:

X_0OY_0 – the two-dimensional inertial system of Cartesian coordinates;

$X_1B_1Y_1$ – the moving coordinates, steadily related to the truck-tractor;

$X_2B_5Y_2$ – the moving coordinates, steadily related to the framework of a semitrailer;

$m_1, I_1; m_2, I_2$ – accordingly, mass and central moment of inertia of the separate modules of a lorry convoy;

v_i, u_i – longitudinal and lateral projections of the speed of the point C_i on the axis of the moving coordinates, steadily related to the modules of a lorry convoy;

ω_i – the angular velocity of a link;

φ_1 – the angle of a lorry convoy folding;

ψ_i – the skew angle of i – th axis of a lorry convoy;

$X_i, X_{ir}, Y_i, Y_{ir}, Z_i, Z_{ir}$ – longitudinal, lateral and vertical reactions of the road to the wheels of i – th axes;

$i = 1$ to 5 – the indexes related to the axes of a lorry convoy;

r – the index related to the starboard of the links of a lorry convoy.

Geometrics of a lorry convoy:

$a_1 = A_1C_1, b_1 = C_1B_1, L_1 = a_1 + b_1, \ c_1 = B_1O_1, h_1 = M_1A_1, \ \varepsilon_1 = M_1N_1, \ b_4 = B_4C_2, b_5 = B_5C_2,$

$a_2 = O_1C_1, h_2 = N_2B_2, h_3 = N_3B_3, h_4 = N_4B_4, h_5 = N_5B_5, b_3 = B_3C_2.$

At the constant motion in the real constructions of automobiles, the difference of angles of external and internal turns in relation to the center of the turn of the guided wheel is the fractions of a degree on

the average. Under such conditions, it is possible to conduct calculations of the middle angle of turn of the wheel $\Theta_{s1} = (\Theta_1 + \Theta_{1r})/2$ with a high degree of accuracy and ignore the redistribution of lateral forces.

In this case the expressions for longitudinal and lateral speed of links of a lorry convoy are written as follows

$$v_2 = v_1;$$
$$u_2 = v_1\varphi_1 + (u_1 - \omega_1(b_1 - c_1)) - (\omega_1 - \dot{\varphi}_1)a_2.$$
(1)

At the rectilinear motion the lateral speeds are far less than longitudinal. At that rate, the middle angles of the skid of axes of a lorry convoy will be written down as

$$\delta_{s1} = -\frac{u_1 + \omega_1 a_1}{v_1}; \qquad\qquad \delta_{s2} = \frac{-u_1 + \omega_1 b_1}{v_1};$$

$$\delta_{s3} = \frac{-v_1\varphi_1 - u_1 - \omega_1(b_1 - c_1) + (\omega_1 - \dot{\varphi}_1)b_3}{v_1}; \quad \delta_{s4} = \frac{-v_1\varphi_1 - u_1 - \omega_1(b_1 - c_1) + (\omega_1 - \dot{\varphi}_1)b_4}{v};$$

$$\delta_{s5} = \frac{-v_1\varphi_1 - u_1 - \omega_1(b_1 - c_1) + (\omega_1 - \dot{\varphi}_1)b_5}{v_1}.$$
(2)

After determining the longitudinal and lateral speeds and accelerations of separate links of a lorry convoy, the angles of skid and lateral forces, affecting the wheels of separate axes (according to Rokar dependence [13]), stabilizing moments of tires, normal reactions of the bearing area on the wheels of all axes, differential equations of the motion of a lorry convoy, taking into account the angles of axle setting, are written as follows

$$(m_1 + m_2)\dot{u} + \dot{\omega}_1 m_2(b_1 - c_1) + m_2 a_2(\dot{\omega}_1 + \ddot{\varphi}_1)\dot{\omega}_0 - v_1\omega_1 = Y_1 + Y_{1r} - m_1\omega_1 v_1 - (X_2 + X_{2r})*$$
$$*\sin\psi_2 + (Y_2 + Y_{2r})\cos\psi_2 + (X_3 + X_{3r})\sin\psi_3 + (Y_3 + Y_{3r})\cos\psi_3 + (X_4 + X_{4r})\sin\psi_4 +$$
$$+ (Y_4 + Y_{4r})\cos\psi_4 + (X_5 + X_{5r})\sin\psi_5 + (Y_5 + Y_{5r})\cos\psi_5;$$
$$m_2(\dot{u}_1 - \dot{\omega}_1(b_1 - c_1) - a_2(\dot{\omega}_1 - \ddot{\varphi}_1) + v_1\omega_1) = I_1\dot{\omega}_1 - (k_1\delta_1 + k_1\delta_{1r})a_1 -$$
$$- (X_1 + X_{1r})*(h_1 + \varepsilon) + X_2(h_2 - b_1\sin\psi_2) - X_{2r}(h_2 + b_1\sin\psi_2) +$$
$$+ (k_2\delta_2 + k_2\delta_{2r})b_1\cos\psi_2 + (X_3 + X_{3r})\sin\psi_3 + (k_3\delta_3 + k_3\delta_{3r})\cos\psi_3 +$$
$$+ (X_4 + X_{4r})\sin\psi_4 + (k_4\delta_4 + k_4\delta_{4r})\cos\psi_4 +$$
$$+ (X_5 + X_{5r})\sin\psi_5 + (k_5\delta_5 + k_5\delta_{5r})\cos\psi_5;$$
$$I_2(\dot{\omega}_1 - \ddot{\varphi}_1) - m_2 a_2(\dot{u} - \dot{\omega}_1(b_1 - c_1) - a_2(\dot{\omega}_1 - \ddot{\varphi}_1) + v_1\omega_1) = -(X_3 + X_{3r})\sin\psi_3 -$$
$$-(k_3\delta_3 + k_3\delta_{3r})\cos\psi_3 - (X_4 + X_{4r})\sin\psi_4 - (k_4\delta_4 + k_4\delta_{4r})\cos\psi_4 - (X_5 + X_{5r})\sin\psi_5 -$$
$$-(k_5\delta_5 + k_5\delta_{5r})\cos\psi_5 + X_3(h_3 - b_3\sin\psi_3) - X_{3r}(h_3 + b_3\sin\psi_3) + X_4(h_4 -$$
$$-b_4\sin\psi_4) - X_{4r}(h_4 + b_4\sin\psi_4) - (k_4\delta_4 + k_4\delta_{4r})b_4\cos\psi_4 + X_5(h_5 -$$
$$-b_5\sin\psi_5) - X_{5r}(h_5 + b_5\sin\psi_5) - (k_5\delta_5 + k_5\delta_{5r})b_5\cos\psi_5.$$
(3)

The acquired system of differential equations (3) describes the motion of a lorry convoy, taking into account the skew of the axles of a truck-tractor and the axes of a semitrailer. The system allows us to investigate the influence of structural parameters and the operating modes of the motion of a lorry convoy on the indexes of the motion road holding.

Under the term critical speed $v_{кр}$ we will consider the speed at which at least one of the links of a lorry convoy loses its stability, which is understood as a property of a lorry convoy link to save the direction of motion and orientation of the longitudinal and vertical axes, without any influences from the side of a driver, in the set limits, regardless of the movement rate and the influence of external forces.

The system of equations of the motion of a lorry convoy presumes the solution $u_1 = 0$, $\omega_1 = 0$, $\varphi_1 = 0$, $(\Theta_1 = \Theta_{1r} = 0)$ [14, 15], the motion of all points of a lorry convoy coincides with this solution at a speed v along the line $\theta = const$ on the planes of a road. We will take such motion as unperturbed.

After the solution of the system of equations (3) with respect to high-order derivatives, we will get

$$\dot{u}_1 = -\frac{m_2 B0 - m_2(b_1 - c_1) + A0m_2C^2I_1 + m_2I_2a_2^2A0 + A0I_2I_1}{m_2C^2m_1I_1 + m_2I_2B^2m + m_2I_1I_2 + m_1I_1I_2}; \tag{4}$$

$$\dot{\omega}_1 = -\frac{m_1m_2(b_1 - c_1)^2B0 + m_2I_2B0 + m_2I_2A0 + I_2m_1B0}{m_2C^2m_1I_1 + m_2I_2B^2m_1 + m_2I_1I_2 + m_1I_1I_2}; \tag{5}$$

$$\ddot{\varphi}_1 = -\frac{m_2(b_1 - c_1)m_1B0 + m_2B^2m_1C0 + m_2C^2m_1B0 + m_2I_2B0}{m_2C^2m_1I_1 + m_2I_2B^2m_1 + m_2I_1I_2 + m_1I_1I_2}, \tag{6}$$

where

$$B0 = \left(\frac{B2}{v_1} - m_2\left(B + 2C\right)v_1\right)\omega_1 + \frac{B1u_1}{v_1} + B3\varphi_1 + \frac{B4\dot{\varphi}_1}{v_1};$$

$$C0 = \left(m_2v_1C + \frac{C2}{v_1}\right)\omega_1 + \frac{C1u_1}{v_1} + C1\varphi_1 + \frac{C3\dot{\varphi}_1}{v_1}. \tag{7}$$

The system of equations in a vector-matrix form looks like

$$\left\|a_{ij}\right\|_1^3 \cdot \begin{vmatrix} \dot{u}_1 \\ \dot{\omega}_1 \\ \ddot{\varphi}_1 \end{vmatrix} + \left\|b_{ij}\right\|_{3,4} \cdot \begin{vmatrix} u_1 \\ \omega_1 \\ \varphi_1 \\ \dot{\varphi}_1 \end{vmatrix} = 0. \tag{8}$$

To find the required and sufficient conditions of stability of the rectilinear motion of an automobile, it is necessary to make up a characteristic equation of the system (3). Partial solution of the system in the vector-matrix form will be found as u_1, ω_1, $\varphi_1 = (a_1,\ a_2,\ a_3)\exp(\lambda t)$ if and only if λ is the root of the characteristic equation

$$D(\lambda) = A_0\lambda^4 + A_1\lambda^3 + A_2\lambda^2 + A_3\lambda + A_4 = 0. \tag{9}$$

The matrix of the characteristic equation in a general view is as follows

$$\begin{vmatrix} a_{11}\lambda + b_{11} & a_{12}\lambda + b_{12} & a_{13}\lambda^2 + b_{13}\lambda + b_{14} \\ a_{21}\lambda + b_{21} & a_{22}\lambda + b_{22} & a_{23}\lambda^2 + b_{23}\lambda + b_{24} \\ a_{31}\lambda + b_{31} & a_{32}\lambda + b_{32} & a_{33}\lambda^2 + b_{33}\lambda + b_{34} \end{vmatrix} = \sum_{i=0}^{n=4} A_i\lambda^{n-i} = 0; \tag{10}$$

where a_{ij} and b_{ij} are the coefficients, determined by the geometrical parameters and the parameters of the mass of a lorry convoy, received analytically in the program Maple 12.

According to Raus-Gurvits criterion of stability [16], the necessary but insufficient condition of stability consists in the need for all coefficients of A_i to be positive. The system will be steady if the determinant and its subdeterminants are positive. The analysis of the roots of the characteristic equation can characterise the state of the system.

In a general case the next values of roots of the characteristic equation are possible: if λ is an actual and positive value, then the system is unsteady, and the motion will be unsteady; if λ is an actual and negative value, then with time the system comes back to a steady position. If the coefficient λ is a complex number, then its positive actual part indicates the presence of divergent oscillations and a negative actual part—the presence of decaying oscillations.

Hurwitz determinants of a characteristic equation (9), namely, the first Δ_1, is responsible for the presence of positive actual roots, and the third Δ_3 for the presence of a positive actual part of the

imaginary complex conjugate roots. From the equation (9) we receive the factors determining the critical speed

$$v_{crit} = f(m_1, m_2, a_1, L_1, c_1, L_2, k_1, k_2, k_3, k_4, ..). \tag{11}$$

Using the expression (11) it is possible to carry out the analysis of constant motion, namely, to define the value of the critical speed of rectilinear motion of a lorry convoy and to ascertain the character of influence on its numeral value of factors, including angles of the alignment of lorry convoy axles.

The calculations of stability indexes are executed for a lorry convoy comprised by the truck-tractor DAF XF 95.430 and the semitrailer KRONE - SDP 24 (categories N3+O4), as the most typical in intercity motor freight activity.

The structure of the characteristic determinant (9) of the system of equations (3) and also the expressions of its coefficients a_{ij}, b_{ij} are so tedious in an analytical form that in the future we will use the general expressions of coefficients A_0, A_1, A_2, A_3, A_4 and the determinant Δ_3 of the characteristic equation. Therefore, we will determine the necessary coefficients with a numeral method by means of a computer design in the environment of Maple 12.

The value of operating and structural parameters of a lorry convoy at which the determinant of the system $A_4 = 0$ will be named critical and at $\Delta_3 = 0$ – flutter. Considering the dynamics of a lorry convoy and taking into account the angles of axle alignment, the basic factor determining the stability of rectilinear motion is a course rate of movement of a lorry convoy. The following situations are typical:

$$A_4 = 0 \Rightarrow v = v_{crit} \; ; \quad A_4 > 0 \Rightarrow v < v_{crit} \; ; \quad A_4 < 0 \Rightarrow v > v_{crit} \; , \tag{12}$$

where v is the traversing speed of a lorry convoy;

$$\Delta_3 = 0 \Rightarrow v = v_0; \quad \Delta_3 < 0 \Rightarrow v > v_0; \quad \Delta_3 > 0 \Rightarrow v < v_0 \; , \tag{13}$$

where v_0 is the tripping speed of the oscillatory instability of the motion of a lorry convoy.

As v_{crit} and v_0 are the functions of the parameters of a lorry convoy, in space of these parameters of the equations $v = v_{crit}$ and $v = v_0$ determine the hypersurfaces on which the characteristic equation (9) has one zero and a pair of complex roots. The first expressions of equations (12) and (13) are possible to write down as the functions of the rate of movement of a lorry convoy:

$$A_4 = f(v_{crit}, \quad other\,factors)$$
$$\Delta_3 = f(v_0, \quad other\,factors) \tag{14}$$

Consequently, there are two characteristic values of rate of movement of a lorry convoy $v = v_{crit}$ and $v = v_0$ that can be received from the equations (14). However, their reduction to the obvious type in connection with a large dimension and the number of incoming parameters is, in a general case, an insoluble problem and does not allow for the use of purely analytical methods of research.

To get the indicated dependencies is possible by means of numerical methods of computer design. As far as obvious expressions of equation solutions (14) are not present, to find the dependencies $v_{crit} = f(A_4)$ and $v_0 = f(\Delta_3)$ we will use the interval method that allows us to calculate any dependencies of the non-obvious kind.

In case of v_{crit} and v_0 for $A_4 = 0$ and $\Delta_3 = 0$ respectively, we have:

$$A_4 = (v, X_i) = 0 \; \text{и} \; \Delta_3 = (v, X_i) = 0 \; , \; i = 1...n, \tag{15}$$

where v − the current value of the speed of a lorry convoy; X_i − the parameters of a lorry convoy;

n − the number of parameters.

Increasing the current speed of a lorry convoy v from v_{min} to v_{max} in $v = (v_{max} - v_{min})/n$, in an interval from v_{min} to v_{max} at every step the conditions $A_4 = 0$ or $\Delta_3 = 0$ are checked up. If one of the conditions is executed, then a current value is assigned to the corresponding one from the extreme speeds v_{crit} or v_0 . Thus, we get the possibility of receiving the dependencies of speeds v_{crit} and v_0

from any parameter of a lorry convoy. Using the basic data for the examined lorry convoy and the developed methodology, we will estimate the critical speed of a rectilinear motion. At the chosen layout diagram of a lorry convoy, the critical speed is about 32,4 m/s (116,6 km/h).

Fig. 2 shows the influence of axis skew of a lorry convoy on the critical speed of a rectilinear motion : 1 – the influence of the skew of the back axis of a semitrailer; 2 – the influence of the skew of the rear axle of a truck-tractor; 3 – the influence of the skew of the second axis of a semitrailer; 4 – the influence of the skew of the first axis of a semitrailer.

The increase of the skew of any axis in any direction diminishes the critical speed of a lorry convoy. The skew of the rear axis of a semitrailer has the most considerable influence. So, the skew of this axis of $0,5^0$ reduces the critical speed of a lorry convoy by 15,1%, the second axle of a truck-tractor by 12,2%, the second and first axis of a semitrailer by 6,2 and 5,1% accordingly. The subsequent increase of the skew over 3° results in the oscillatory instability of a lorry convoy.
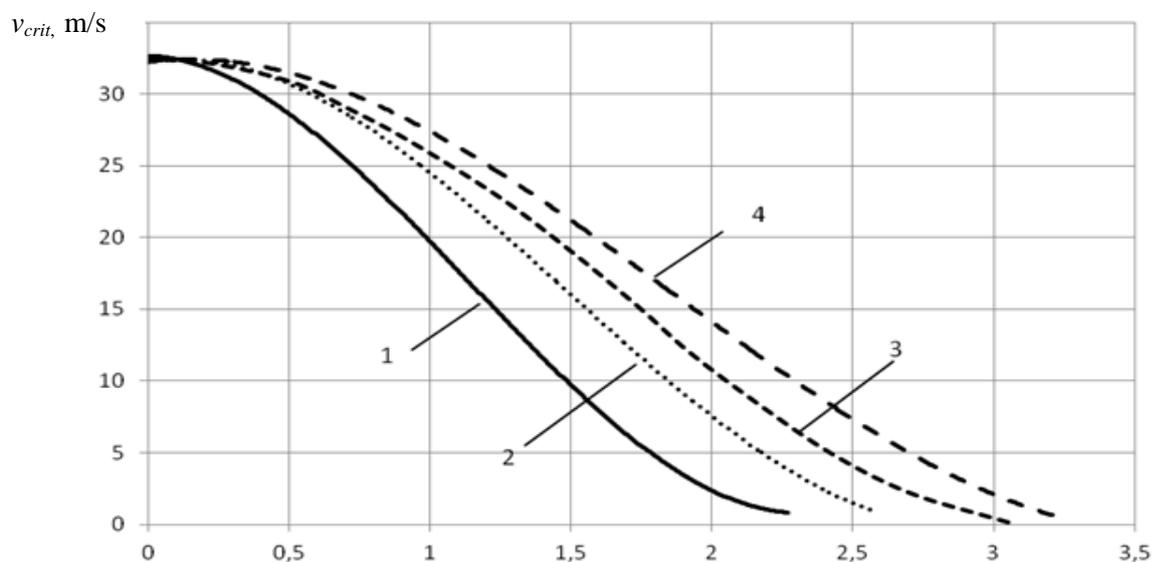
Fig. 2. Changing of the critical speed of the rectilinear motion of a lorry convoy due to the skew ⟨

Рис. 2. Изменение критической скорости прямолинейного движения автопоезда от перекоса его осей

4. CONCLUSION

The mathematical model of the motion of a lorry convoy is worked out taking into account the angles of the alignment of the axles of a truck-tractor and the axes of a semitrailer. It is shown that the motion of a lorry convoy without the skew of the axes is asymptotically steady. Critical speed of rectilinear motion of a lorry convoy consisting of the saddle-type tractor DAF XF 95.430 and the semitrailer KRONE - SDP 24 is determined. This critical speed without the skew of the axles of a truck-tractor and the skew of the axes of a semitrailer is 32,4 m/s (116,6 km/h). In case of the skew of the second axle of a truck-tractor of $0,5^0$, the critical speed of the rectilinear motion reduces to 30,2 m/s (108,5 km/h).

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