fastening, wheel machinery, constraint reaction,
friction force, loaded wagon vibrations

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DEVELOPMENT OF DYNAMICAL MODEL OF WHEEL MACHINERY ALLOCATED ON A FLAT–CAR

Summary. The paper gives the results of dynamical modeling of the mechanical system “flat car – elastic elements – wheel machinery body”, allocated on a railway flat car. There have been obtained the formulas of equivalent rigidity of fastening spatial flexible elements relative to a vertical line, which are equal to rigidity of bus and spring flexible elements being plugged in series and which are then equal to rigidity of all elastic elements of wheeled machinery as springs being plugged in parallel.

ПОСТРОЕНИЕ ДИНАМИЧЕСКОЙ МОДЕЛИ КОЛЁСНОЙ ТЕХНИКИ, РАЗМЕЩЁННОЙ НА ЖЕЛЕЗНОДОРОЖНОЙ ПЛАТФОРМЕ

Аннотация. В статье приведены результаты построения динамической модели механической системы «платформа – упругие элементы – кузов колёсной техники», размещённой на железнодорожной платформе. Получены формулы эквивалентной жёсткости пространственно расположенных гибких элементов креплений по вертикали, эквивалентные жёсткости последовательно включённых упругих элементов шин и рессор, а затем эквивалентные жёсткости всех упругих элементов колёсной техники, как параллельно включённых пружин.

1. FORMULATION OF A PROBLEM

It is well-known [1, 2], that on open rolling-stock, cargo (both solid and wheeled) is prevented from displacement relative to wagon floor with the help of the following transportation safety devices (fastenings): flexible elastic (fastenings and bindings) and thrust (wooden thrust and cross-bars) elements. In which connection, thrust elements of fastenings are fastened to the wagon floor with fixing wares (nail) tightly to frontal and lateral cargo surfaces, e. g., to machinery wheels (fig. 1). At that, weakening (sagging) of flexible elastic elements of other fastenings and fastening of thrust bars to the wagon floor at some distance from the wheel machinery are not allowed according to Specs.

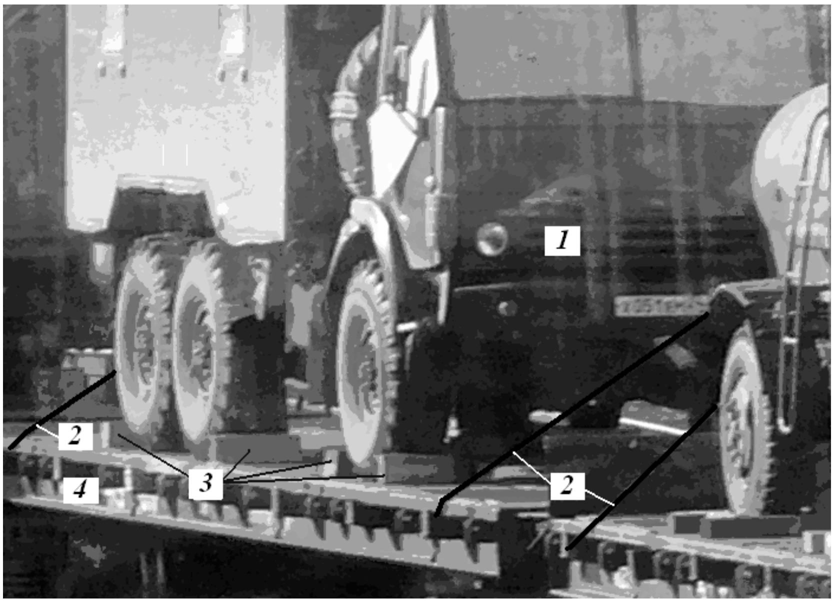


Fig. 1. Allocation and fastening of wheel machinery on a flat car: 1 – wheel machinery; 2 – fastening flexible elements; 3 – thrust bars; 4 – flat car
Рис. 1. Размещение и крепление колёсной техники на платформе: 1 – колёсная техника; 2 – гибкие элементы креплений; 3 – упорные бруски; 4 – платформа

It should be borne in mind while elaborating technology of cargo fastening in a wagon that fastening disorders en route may occur due to imperfection of their calculation techniques where many real factors are not taken into account, such as pressure reduction in wheels, fastening wire preload etc.

These factors make a considerable impact on their conveyance en route. For instance, reduction of pressure in wheels may occur en route, thus resulting in fastening disorder as flexible element sagging (weakening) (fig. 2), that lead to wheel subsidence and lowering of the waggoned machinery body (fig. 3). In which connection, disorder of wheeled machinery allocation relative to the flat car floor may occur, resulting in creation of potentially hazardous situations while conveyance.

Fig. 2. Weakening of fastening flexible elements of wheel machinery
Рис. 2. Ослабление гибких элементов креплений колёсной техники

Fig. 3. Pressure reduction of wheel machinery wheels on a flat car
Рис. 3. Снижение давления колёс колёсной техники на платформе

In [1] we considered interaction of the open rolling-stock and rigid cargo as simplified, found out causes of cargo displacement (shift) relative to the wagon floor. The results were set out concerning elaboration of scientifically grounded rational technology of allocation and fastening of cargo in wagons from planar system of forces, and in [2] – of spatial system of forces, contributing to assurance of movement safety and cargo undamaged state. But until present day, elaboration of technology of allocation and fastening of wheel machinery on a flat car through calculation (dynamic) and mathematical modeling of conveyance of such cargo remains scantily known or completely unexplored.

1.1. Man-made assumption

We need dynamical modeling of the wheel machinery, allocated on a flat car. While solving problems involving relative motion of wheeled machinery on a flat car, there appears the necessity in rigidity of fastening flexible elements located in the space arbitrarily, either relative to a longitudinal axis, or a transverse axis, or a vertical axis.

We should consider one of the ways to determine equivalent rigidity of fastening flexible elements $c_{ekv.\mathrm{z}}$ relative to the flat car vertical axis [3, 4].

We assume that the wheel machinery (further – WM) is held fixed relative to the flat car with a pair of fastening flexible elastic elements as is shown in fig. 4.

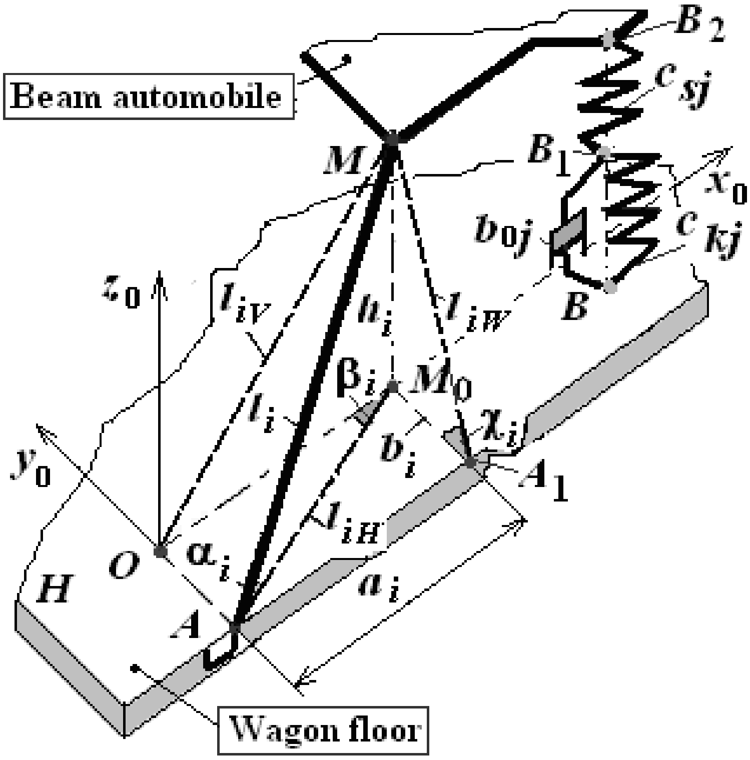


Fig. 4. Geometry of a fastening flexible elastic element
Рис. 4. Геометрия гибкого упругого элемента крепления

Indicated in fig. 4: A – flat car frame bracket; M – cargo ear of wheeled machinery (WM); AM – WM flexible elastic element; l_i – fastening element length; a_i , b_i and h_i – projections of fastening elastic elements on transverse, longitudinal and vertical axes; i = 1, n_p – number of WM fastening flexible elastic elements; α_i – angle between the elastic element length and its projection l_{iH} on the car floor plane; β_i – angle between elastic element projection on the car floor plane and longitudinal axis Ox; B – contact point of WM wheels with car floor; B_1 – WM wheel axis (or connection between spring and WM wheel axis); B_2 – points on WM frame where springs are connected with frame; c_{kj} and b_0 – coefficients of rigidity and WM bus viscous resistance; c_{sj} – WM spring rigidity coefficient; j = 1, n_k – number of WM wheels and springs.

We can show derivation of the formula to determine equivalent rigidity of the fastening flexible elastic element under action of vertical forces on the wheel machinery.

The wheel machinery allocated on the flat car is kept from displacement by fastening wire flexible elements with rigidities c_i, fixed at points I with one end at the cargo ears (or hook) of wheel machinery, e.g., at point M (fig. 5) and with the other end – at flat car framed brackets, e.g., at point A.

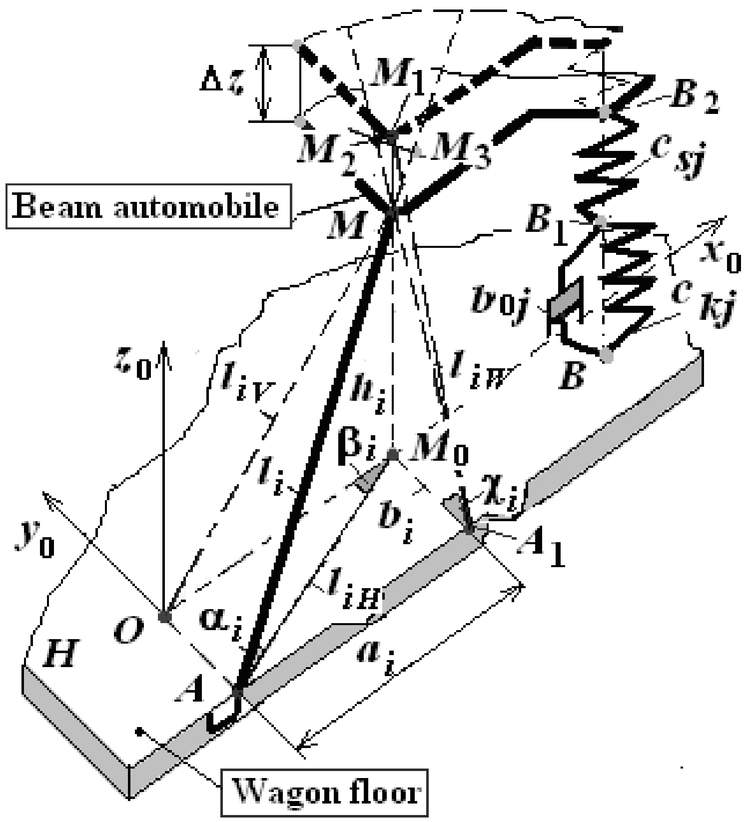


Fig. 5. Lengthening of fastening elastic element relative to vertical
Рис. 5. Удлинение гибкого упругого элемента крепления по вертикали

In order to determine rigidity coefficient of fastening elements within terminal displacement of wheel machinery relative to *Oz* axis, we shall bear in mind that the length of the fastening element A_1M which is shown on the estimated model is the projection of its initial length *AM* on the frontal plane *W*, forming the angle χ_i relative to the wagon transverse axis *Oy*, determined according to the formula

$$\sin\chi_i = \frac{h_i}{l_{iW}}, \tag{1}$$

where: b_i – projection of fastenings on the longitudinal axis *Oy*; l_{iW} – projections of fastenings on the frontal plane *W*.

The length of fastening flexible elastic element l_i forms together with its projection on the vertical plane the angle ($\pi/2 - \beta_i$), which is found according to the formula

$$\cos\left(\frac{\pi}{2} - \beta_i\right) = \sin\beta_i = \frac{l_{iW}}{l_i}. \tag{2}$$

We assume that the vertical displacement of the wheeled machinery will occur relative to the flat car by the value $\Delta z = z$ under external disturbance. We believe that the cargo fixing point *M* will take the position M_1, and the projection of the fastening element length on the frontal plane *W* will become equal to $A_1M_1 > A_1M$.

2. METHODS OF SOLUTION AND RESULTS

To solve the assigned task we used the technique of double projection of arbitrary spatial forces, which is known in Theoretical Mechanics [3, 4].

According to the method of determination of deformations in case of minor displacement, a new position of a wheeled machinery fixing point, i. e. $M_{1,}$ is projected on "initial" or "old" direction of the elastic element. Inasmuch as the elastic element is located in the space arbitrarily, in order to calculate the projection it is necessary to apply a method of double projection in the way it is used in Theoretical Mechanics for an arbitrarily located force [3, 4], i. e, at first perpendicular M_1M_2 is put from the point M_1 on the continuation of the line A_1M, and then – M_2M_3 on continuation of the line AM.

In order to determine terminal displacement of the point M from ΔMM_1M_2 and ΔMM_2M_3, we should find the projection of wheel machinery vertical displacement $\Delta z = z$ on "initial" direction of fastening flexible elements with consideration of wire pretwisting of each separate fastening element. At the same time we take into account that $\angle M_1MM_2 = \angle M_0MA_1 = \left(\frac{\pi}{2} - \chi\right)$, but $\angle M_2MM_3 = \angle A_1MA = \angle A_1M_0A = \left(\frac{\pi}{2} - \beta\right)$. In accordance with that, we have

$$MM_{2i} = \Delta z\cos\left(\frac{\pi}{2} - \chi_i\right) = \Delta z\sin\chi_i;\ MM_{3i} = MM_{2i}\cos\left(\frac{\pi}{2} - \beta_i\right) = MM_{2i}\sin\beta_i.$$

After putting the previous expression into the latest one, we receive

$$MM_{3i} = \Delta z\sin\chi_i\sin\beta_i, \tag{3}$$

or with consideration of formulas (1) and (2) and $\Delta z = z$, we have

$$MM_{3i} = z\frac{h_i}{l_i}. \tag{3a}$$

Elastic force $\overline{F}_{elast}$ ($F_{elast.x}$, $F_{elast.y}$ and $F_{elast.z}$) varies depending on length change (deformation) of spring linkage (of fastening flexible elements). Elastic force value is calculated according to Hooke's law [3, 4]

$$F_{elast.i} = -c_i\Delta l_i, \tag{4}$$

where: Δl_i – lengthenings of the elastic element i relative to its length, m; $\boldsymbol{c_i}$ – rigidity coefficient of the fastening elastic element i as ratio of rigidity per stretching of the flexible elastic element to its length, kN/m:

$$c_i = \frac{EA_i}{l_i}, \tag{5}$$

EA_i – physical and geometrical feature (rigidity per stretching) of the flexible elastic element, kN;

E – module of rigidity of the flexible elastic element material, twisted from the steel annealed wire ($E = 1\cdot 10^7\ kN/m^2$);

A_i – cross-section area of the flexible elastic element i, m^2

$$A_i = n_i\frac{\pi 10^{-6}d_i^2}{4}, \tag{6}$$

with consideration that n_i – number of cords in i – elastic flexible element; d_i – wire diameter of the flexible elastic element (mm);

l_i – length of the flexible elastic element, m:

$$l_i = \sqrt{a_i^2 + b_i^2 + h_i^2}, \tag{7}$$

where: a_i, b_i and h_i – projections of fastening flexible elements on coordinates, m (see fig. 5).

With consideration of (6) we configure the equation (5)

$$c_i = \frac{\pi 10^{-6}E}{4}d_i^2\frac{n_i}{l_i}. \tag{8}$$

Expression (4) is referred to physical equations which connect force and displacement.

Stretching (effort) in the flexible elastic element with consideration of pretwisting tension is determined by the formula [3, 4]

$$R_i = c_i MM_{3i} + R_{0i}\,,$$

or, considering (3, a),

$$R_i = c_i z \frac{h_i}{l_i} + R_{0i}\,, \tag{9}$$

where: R_{0i} – pretwisting response of each flexible elastic element, *kN* (20 *kN* may be accepted).

If we project stretching in fastening elements R_i on a horizontal plane H, we have

$$R_{Hi} = R_i \cos\alpha_i\,. \tag{10}$$

We determine projections of stretching (elastic force) in fastening elements on longitudinal, transverse and vertical axes:

$$R_{xi} = R_{Hi}\cos\beta_i\,,\ R_{yi} = R_{Hi}\sin\beta_i\,;\ R_{zi} = R_i \sin\alpha_i\,.$$

or with consideration (10) we have

$$R_{xi} = R_i \cos\alpha_i \cos\beta_i\,,\ R_{yi} = R_i\cos\alpha_i \sin\beta_i\,;\ R_{zi} = R_i\sin\alpha_i\,, \tag{11}$$

where

$$\cos\alpha_i\cos\beta_i = \frac{a_i}{l_i},\quad \cos\alpha_i\sin\beta_i = \frac{b_i}{l_i},\quad \sin\alpha_i = \frac{h_i}{l_i}. \tag{11a}$$

In accordance with this, we configure correlations (11)

$$R_{xi} = R_i\frac{a_i}{l_i}\,,\qquad R_{yi} = R_i\frac{b_i}{l_i}\,;\quad R_{zi} = R_i\frac{h_i}{l_i}\,, \tag{12}$$

Putting (9) into the latest equations, we have

$$R_{xi} = \left(c_i z\frac{h_i}{l_i} + R_{0i}\right)\frac{a_i}{l_i}\,,\ R_{yi} = \left(c_i z\frac{h_i}{l_i} + R_{0i}\right)\frac{b_i}{l_i}\,;\ R_{zi} = \left(c_i z\frac{h_i}{l_i} + R_{0i}\right)\frac{h_i}{l_i}\,, \tag{13}$$

With consideration (8) we may configure the latest correlations, which characterize projections of stretching in wheel machinery fastening elements:

$$R_{xi} = \left(\frac{\pi 10^{-6} E d_i^2}{4}\frac{n_i}{l_i} z\frac{h_i}{l_i} + R_{0i}\right)\frac{a_i}{l_i};\qquad R_{yi} = \left(\frac{\pi 10^{-6} E d_i^2}{4}\frac{n_i}{l_i} z\frac{h_i}{l_i} + R_{0i}\right)\frac{b_i}{l_i};$$

$$R_{zi} = \left(\frac{\pi 10^{-6} E d_i^2}{4}\frac{n_i}{l_i} z\frac{h_i}{l_i} + R_{0i}\right)\frac{h_i}{l_i}.$$

Taking into account the impact of vertical transient inertia force on wheel machinery – I_{ez}, gravitational components – G_z and wind resistance forces – F_b, taken by fastening elastic elements i,

$$F_x = \sum_{i=1}^{n} R_{xi} = \sum_{i=1}^{n}\frac{\pi 10^{-6} E d_i^2}{4}\frac{n_i}{l_i}\frac{a_i}{l_i}\frac{h_i}{l_i} z + \sum_{i=1}^{n} R_{0i}\frac{a_i}{l_i};$$

$$F_y = \sum_{i=1}^{n} R_{yi} = \sum_{i=1}^{n}\frac{\pi 10^{-6} E d_i^2}{4}\frac{n_i}{l_i}\frac{b_i}{l_i}\frac{h_i}{l_i} z + \sum_{i=1}^{n} R_{0i}\frac{b_i}{l_i};$$

$$F_z = \sum_{i=1}^{n} R_{zi} = \sum_{i=1}^{n}\frac{\pi 10^{-6} E d_i^2}{4}\frac{n_i}{l_i}\frac{h_i}{l_i}\frac{h_i}{l_i} z + \sum_{i=1}^{n} R_{0i}\frac{h_i}{l_i}.$$

We may rewrite the latest equations as

$$F_x = \sum_{i=1}^{n} R_{xi} = c_{ekv.x} z + \sum_{i=1}^{n} R_{0i}\frac{a_i}{l_i};\quad F_y = \sum_{i=1}^{n} R_{yi} = c_{ekv.y} z + \sum_{i=1}^{n} R_{0i}\frac{b_i}{l_i};$$

$$F_z = \sum_{i=1}^{n} R_{zi} = c_{ekv.z} z + \sum_{i=1}^{n} R_{0i}\frac{h_i}{l_i}\,,. \tag{14}$$

where: $c_{ekv.x}$, $c_{ekv.y}$ and $c_{ekv.z}$ – equivalent (or modified, or generic) rigidities of fastening flexible elastic elements relative to longitudinal *x*, transverse *y* and vertical *z* axes, *kN*/*m*:

$$c_{ekv.x} = \frac{\pi 10^{-6} E}{4} d_i^2 \sum_{i=1}^{n} \frac{n_i}{l_i} \frac{a_i}{l_i} \frac{h_i}{l_i}; \; c_{ekv.y} = \frac{\pi 10^{-6} E}{4} d_i^2 \sum_{i=1}^{n} \frac{n_i}{l_i} \frac{b_i}{l_i} \frac{h_i}{l_i};$$

$$c_{ekv.z} = \frac{\pi 10^{-6} E}{4} d_i^2 \sum_{i=1}^{n} \frac{n_i}{l_i} \frac{h_i}{l_i} \frac{h_i}{l_i}. \tag{15}$$

Therefore, we received analytical formulas of equivalent rigidities of fastening flexible elastic elements relative to longitudinal, transverse and vertical axes for displacement of wheel machinery relative to a wagon vertical with consideration of physical and geometrical features of elastic elements (i. e. E, n, d, l).

As a result of the performed analysis we shall obtain intermediate model of wheel machinery allocated on a flat car (fig. 6)

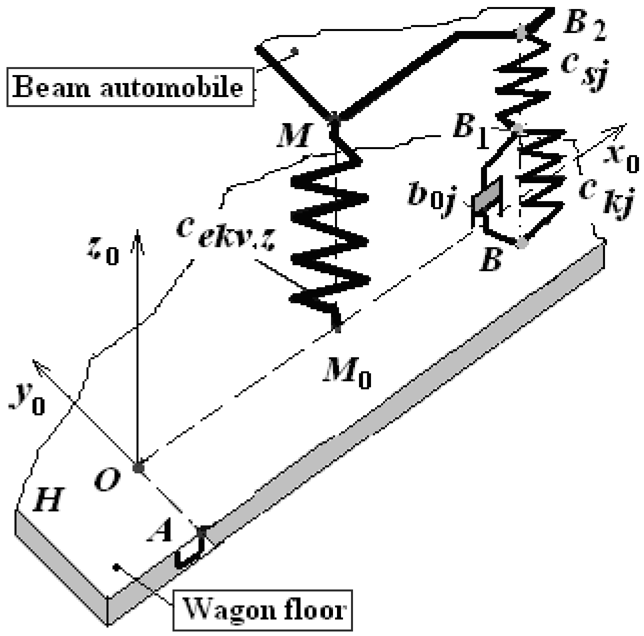


Fig. 6. Intermediate model of wheel machinery allocated on a flat car
Рис. 6. Промежуточная модель колёсной техники, размещённой на платформе

We should determine equivalent rigidities of all wheel machinery vertically bearing in mind the rule of calculation of equivalent rigidity of elastic elements (springs) [3] connected in series and in parallel.

Equivalent rigidities of bus and spring we calculate as springs being connected in series, *kN*/*m* [3] (fig. 7):

$$c_{ks} = \sum_{j=1}^{n_k} \frac{c_{kj} c_{sj}}{c_{kj} + c_{sj}}. \tag{16}$$

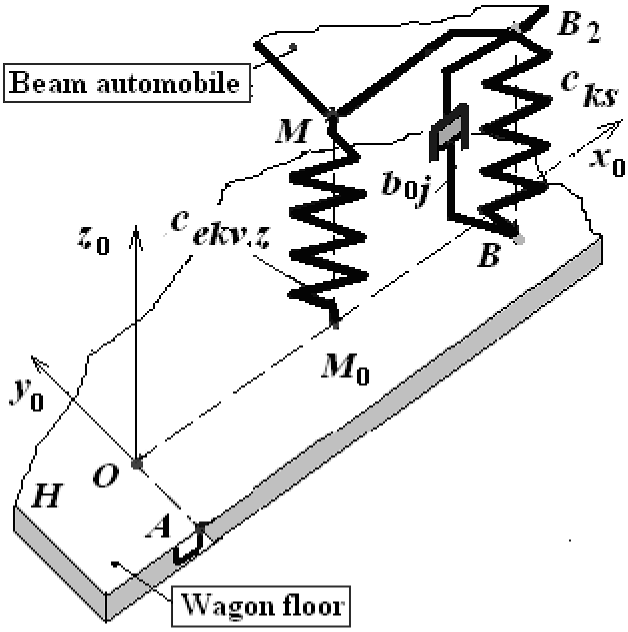


Fig. 7. Intermediate model of wheel machinery allocated on a flat car
Рис. 7. Промежуточная модель колёсной техники, размещённой на платформе

Equivalent rigidities of all wheel machinery vertically (of buses, springs and flexible fastening elements) we find as springs connected in parallel, *kN/m* [3] (see Fig. 7):

$$c_{ekv} = c_{ekv.z} + c_{ks}. \tag{17}$$

Thus, we obtain the dynamic model of wheel machinery allocated on a flat car, as single-mass mechanical system “flat car – elastic elements – wheel machinery body” upon a vibrating base (fig. 8). Here elastic elements are meant as flexible elastic elements of fastenings, wheels and WM springs

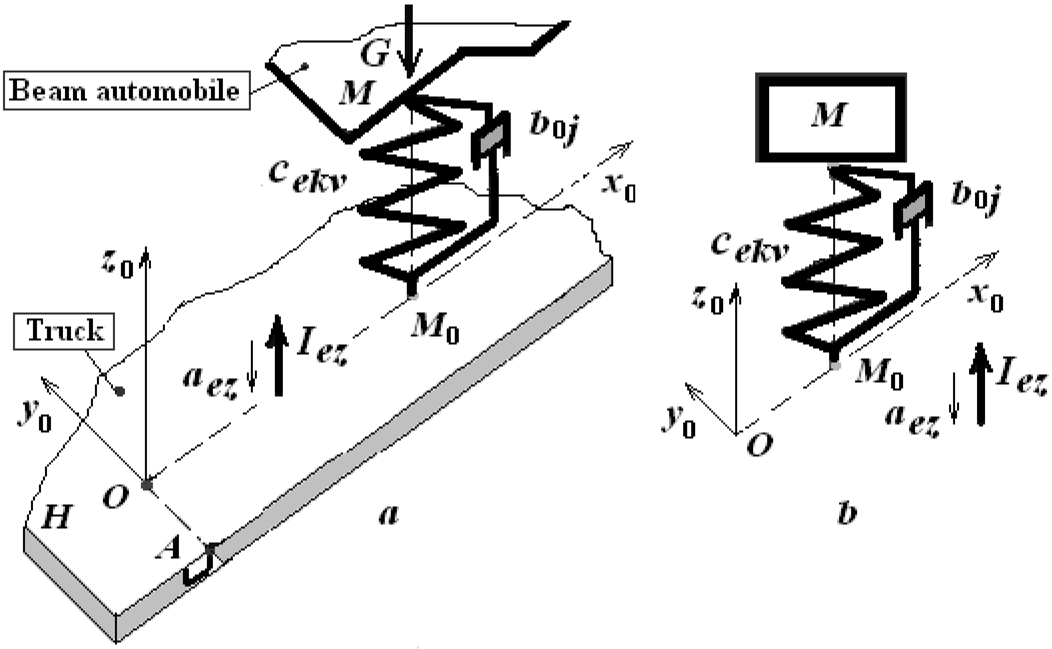


Fig. 8. Dynamical modeling of wheel machinery: *a* – WM body, connected with flat car by elastic element and viscous environment; *b* – single-mass WM model on vibrating base
Рис. 8. Динамическая модель колёсной техники: *a* – кузов КТ, связанный с платформой упругим элементом и вязкой средой; *б* – одномассовая модель КТ на колеблющемся оснований

In Fig. 8 are indicated: G – force of gravitation of WM; a_{ez} – transient acceleration of a flat car provoked by a wave of track irregularity; I_{ez} – transient inertia force.

We should emphasize that standard value of transient acceleration a_{ez} ranges from 0,46 to 0,66g and respectively, the value of transient inertia force equals I_{ez} = (0,46 – 0,66) G [1, 2, 5].

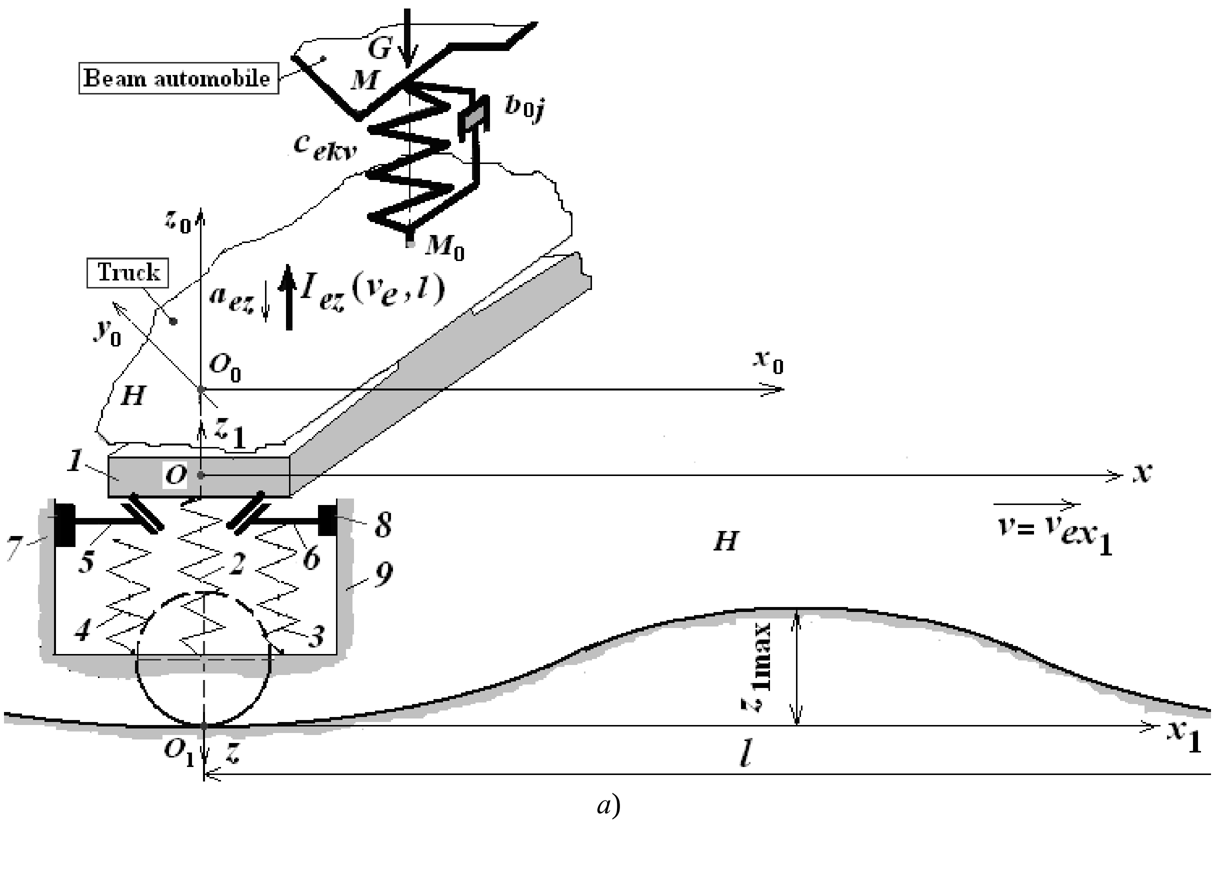
Thus, dynamical modeling of mechanical system "flat car – elastic elements – wheel machinery body" is accomplished.

In particular case, when axes of front and rear wheels of wheeled machinery rest upon special supports which are rigidly connected with flat car floor, coefficients of rigidity and bus viscous resistance are excluded from estimated model, as $c_{kj} = 0$ and $b_{0j} = 0$.

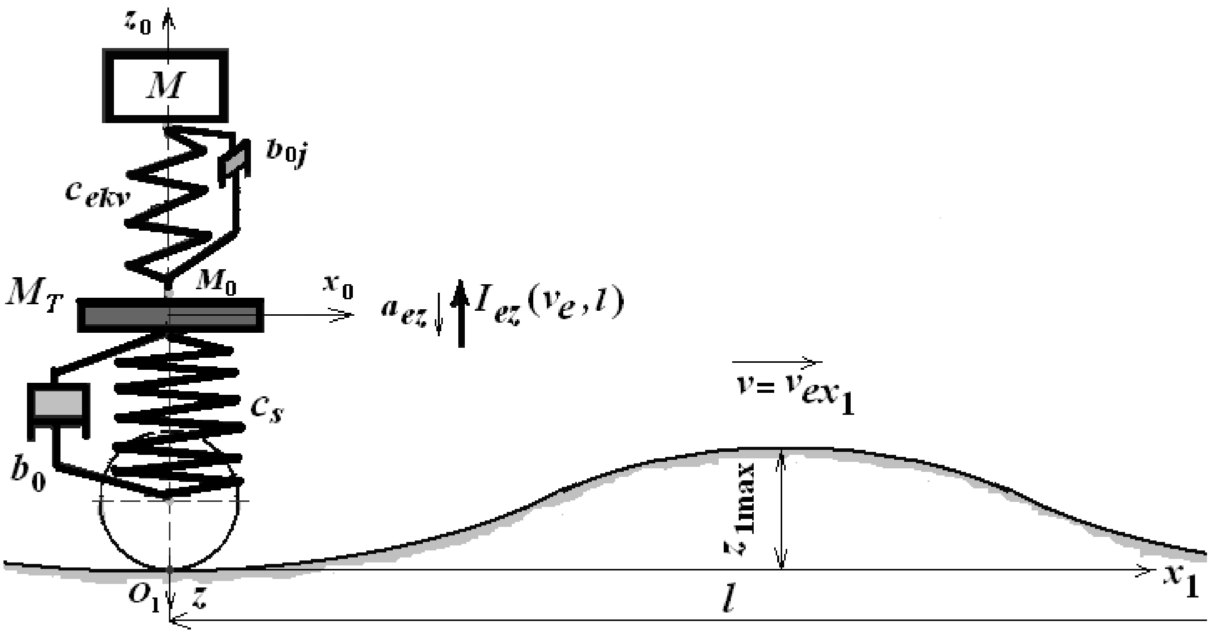
Then we may configure (16) as

$$c_{ks} = \sum_{j=1}^{n_k} c_{sj}. \tag{16a}$$

Taking into consideration that a flat car is exposed to vibrations from a wave of track irregularity, dynamic model of single-mass vibrating system (see fig. 8) can be presented as two-mass vibrating system (fig. 9).



a)



b)

Fig. 9. Mechanical system “single-axle vehicle – flat car – wheel machinery” *a*) –dynamical modeling of wheel machinery and flat-car as single-axle vehicle with dry friction force; *b*) – two-mass vibrating system with elastics elements and viscous media

Рис. 9. Механическая система «одноосный экипаж – платформа – колёсная техника». *а*) – динамическая Модель колёсной техники и платформы, как одноосного экипажа с сухим трением; *б*) – двухмассовая колебательная система с упругими элементами и вязкими средами

In Fig. 9, *a* are indicated: *1* – body of the wagon with cargo; *2* – *4* – elastic elements; *5* – *6* – frictional wedges; *7* – *8* – frictional planks; *9* – truck solebars. In fig.9, *b* dry friction in flat-car spring sets is replaced by the resistance of viscous medium [6].

3. SUMMARY

Summing up the results of the performed analysis we can note there have been obtained the formulas of equivalent rigidity of fastening spatial elastic elements relative to a vertical line, which are equal to rigidity of bus and spring flexible elements being plugged in series and which are then equal to rigidity of all flexible elements of wheel machinery as springs being plugged in parallel. The dynamic model of mechanical system “flat car – elastic elements – wheel machinery body” allocated on a flat car which is exposed to a track irregularity wave is the basis for derivation of equations of vibrations relative to vertical line of wheel machinery.

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