Crane load, dynamic coefficient, track roughness

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# REMARKS ON METHODS OF CALCULATING THE CRANE LOAD CAUSED BY RUNNING ONTO THE TRACK ROUGHNESS

**Summary.** The article considers the single degree of freedom model of crane during its passage through the track roughness (the rail step). The influence of various forms of kinematic excitation to the value of the dynamic coefficient is studied.

# UWAGI NA TEMAT METODY OBLICZANIA OBCIĄŻEŃ DŹWIGNIC WYWOŁANYCH NAJAZDEM NA NIERÓWNOŚĆ TORU

**Streszczenie.** W artykule rozważano model suwnicy o jednym stopniu swobody podczas jej przejazdu przez nierówność toru (próg), badając wpływ różnych postaci wymuszenia na wartość współczynnika dynamicznego.

## 1. INTRODUCTION

In recent years, several publications on the principles of calculating the dynamic coefficients used to determine the load-bearing structures of the crane has emerged in the Polish literature. In the works [1, 2] provision of introduced standard [7], now superseded by [8], were analyzed comparing it with previous methods of calculation. In [3] the attention to the effects of contact rigidity of a running wheel on load-bearing structures' vibration (especially in the case of the use of other than steel wheels) was drawn. Drawbacks of the single degree of freedom model used in norms [7, 8] were also pointed out. The base frequency of system's vibration determined experimentally differ significantly from that calculated for the model.

Authors [4] criticized the movement trajectory shape of wheel overcoming track roughness from norms [7, 8], and then proposed [5] a different method of calculating the dynamic coefficient, based on the assumption of the quasi-rectangular pulse acceleration.

Because norms [7, 8] does not explain all the details of models used, below an attempt was made to systematize issues relating to methodology of calculating the dynamic coefficient associated with the phenomenon of overcoming track roughness. At this stage limitation to the single degree of freedom model was made.

## 2. SINGLE DEGREE OF FREEDOM MODEL

Based on Fig. 1, assuming that the coordinate *z(t)* describes the vibrations around the static equilibrium position, and *h(t)* is the function of roughness elevation, the equation of motion can be written:

$$m\ddot{z} = F(t) - mg \; ; \qquad F(t) = c[h(t) - z(t) + z_{stat}] \; ; \; cz_{stat} = mg \; ; \tag{1}$$

where: $m$ – reduced mass of bearing structure with the load; $c$ – equivalent stiffness of the system.

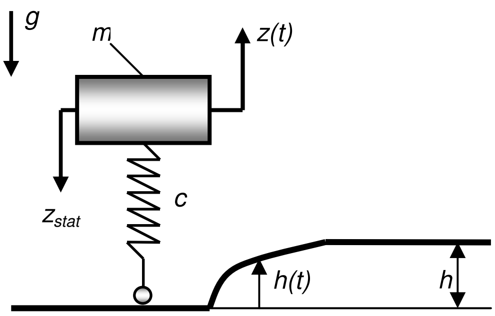


Fig. 1. The single degree of freedom model
Rys. 1. Model o jednym stopniu swobody

After transformation and substitution of $\omega^2 = c/m$ the following equation is received:

$$\ddot{z} + \omega^2 z = \omega^2 h(t) \tag{2}$$

The solution of equation (2) with zero initial conditions can be presented in the form of Duhamel integrals:

$$\begin{aligned} z(t) &= \omega \int_0^t h(\tau) \sin \omega(t-\tau) d\tau && t \le t_s \\ z(t) &= \omega \int_0^{t_s} h(\tau) \sin \omega(t-\tau) d\tau + \omega h \int_{t_s}^{t} \sin \omega(t-\tau) d\tau && t > t_s \end{aligned} \tag{3}$$

where collision time $t_s$ can be estimated as a time of overcoming by wheel centre of crane of the distance $e$ (Fig.2) with velocity $V$, *i.e.* $t_s = e/V$. From the Fig.2 it also follows that: $e = \sqrt{2Rh - h^2} \approx \sqrt{2Rh}$ ; $\sin\alpha = e/R$. In the case of the round step (Fig.3) $R' = R + r$ [5] should be substituted.

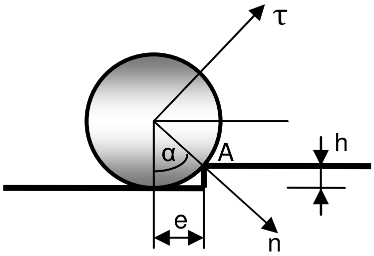


Fig. 2. Sharp step
Rys. 2. Ostry próg

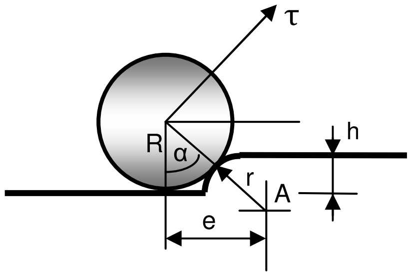


Fig. 3. Round step
Rys. 3. Zaokrąglony próg

According to the norm and accepted indications (Fig.1), the dynamic coefficient is equal to:

$$\varphi_4 = \frac{mg + m|\ddot{z}_{\min}|}{mg} = 1 + \Phi; \quad \Phi = \frac{|\ddot{z}_{\min}|}{g}; \tag{4}$$

where: $\ddot{z}_{\min}$ - the smallest (negative) value of acceleration acting on reduced mass of crane; $\Phi$ - auxiliary dynamic coefficient used for further comparisons.

## 3. KINEMATIC EXCITATION ACCORDING TO THE NORM

Elevation function according to norms [7,8] is described by relation:

$$h(t) = \frac{h}{2}(1 - \cos\omega_s t) \quad t \le t_s; \quad h(t) = h \quad t > t_s; \quad \omega_s = \pi / t_s \tag{5}$$

Substituting to $(3)_1$ and integrating for time $t<t_s$, it is obtained:

$$\begin{aligned} z(t) &= \frac{h}{2}\left(1 + \frac{\omega^2}{\omega_s^2 - \omega^2}\cos\omega_s t - \frac{\omega_s^2}{\omega_s^2 - \omega^2}\cos\omega t\right) \quad \omega \ne \omega_s \\ z(t) &= \frac{h}{2}\left(1 - \cos\omega t - \frac{1}{2}\omega t \sin\omega t\right) \quad \omega = \omega_s \end{aligned} \tag{6}$$

Solution for time $t>t_s$ can be obtained directly by integrating $(3)_2$ or by superposition method used for solutions (6):

$$\begin{aligned} z(t) &= h\left\{1 - \frac{1}{2}\frac{\omega_s^2}{\omega_s^2 - \omega^2}[\cos\omega(t - t_s) + \cos\omega t]\right\} \quad \omega \ne \omega_s \\ z(t) &= h\left(1 - \frac{1}{4}\pi \sin\omega t\right) \quad \omega = \omega_s \end{aligned} \tag{7}$$

To evaluate the dynamic coefficient it is indispensible to know the acceleration. It can be directly determined from formula (2):

$$\ddot{z} = \omega^2[h(t) - z(t)] \tag{8}$$

Hence for $t>t_s$ in case $(7)_1$:

$$\ddot{z} = \frac{\omega^2 h}{2}\frac{\omega_s^2}{\omega_s^2 - \omega^2}[\cos\omega(t - t_s) + \cos\omega t] \tag{9}$$

Acceleration extremum can be determined in classical manner, by searching for zero points of derivative of function (9). Extremum will occur for time instant for which:

$$tg\,\omega t = \frac{\sin\omega t_s}{1 + \cos\omega t_s}; \quad \omega t_s \ne \pi(1 + 2k), \; gdy \; \omega t_s = \pi(1 + 2k) \; to \; \ddot{z} = 0 \tag{10}$$

Another method is appropriate transformation of trigonometric functions in formula (9) and determining the amplitude of function $z(t)$ changes. For both cases it is obtained that:

$$z_{extr} = \pm \frac{h}{2} \frac{\omega^2 \omega_s^2}{\omega_s^2 - \omega^2} \sqrt{2 + 2\cos \omega t_s} = \pm h \frac{\omega^2 \omega_s^2}{\omega_s^2 - \omega^2} \left| \cos(\frac{\pi}{2} \frac{\omega}{\omega_s}) \right| \tag{11}$$

where: $\omega / \omega_s$ corresponds to coefficient $\alpha_s$ introduced in norms [7,8]. Transforming (11) the formula for coefficient $\xi_s$, from appendix D in norm [7], can also be obtained. In opposition to the norm, taking into account the possibility of sign change in (11) protects from receiving negative values of dynamic coefficient $\Phi$. Value $\omega_s$ represents collision parameters: $\omega_s = \pi V / \sqrt{2Rh}$ and can change in a quite wide range. Meaningful influence of velocity *V* on value $\omega_s$ can be verified.

Exemplary graph of dimensionless function $z_{extr} / g$ depending on travel velocity *V,* for fixed parameters (frequency of system's free vibration *f=10Hz*, $\omega = 2\pi f$, *R=0.2m*, *h=0.001m*), is shown on Fig. 4. For small velocities function is not monotonic (pulsation is visible) what can make exact evaluation of dynamic coefficient difficult. That part of the graph corresponds to big values of $\alpha_s$. However, usage of models included in norm is restricted only for $\alpha_s < 1.3$.

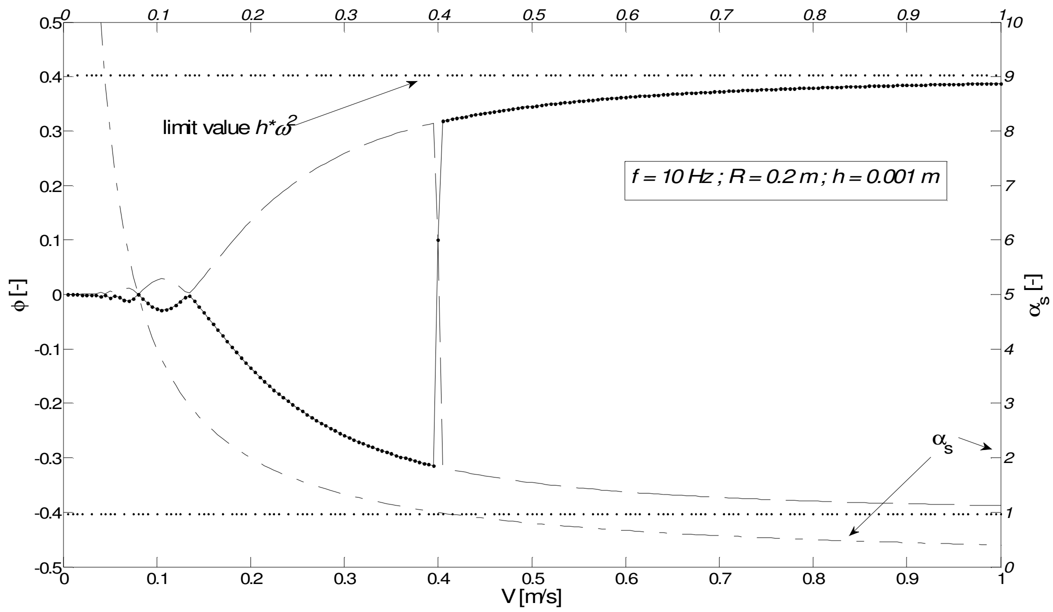


Fig. 4. Exemplary dependence of dynamic coefficient from the speed of the travel of crane
Rys. 4. Przykładowa zależność współczynnika dynamicznego od prędkości jazdy suwnicy

From the graph (Fig.4) it can be seen also that function $z_{extr}$ tends to limit value: $\lim_{\omega_s \to \infty} z_{extr} = \pm h\omega^2$, depending only on step height and frequency of system's free vibration. Lack of monotonicity and limit values occurrence is visible also on Fig. 5.

Extreme value of acceleration may also occur (Fig. 6) while acting the excitation ($t<t_s$). However, in this case, it is hard to present a closed mathematical solution. The dependence of the time of minimum acceleration values was determined numerically. Discrete values were taken $h \in \{1,2,3\}\,[mm]$ and $2R \in \{200,315,400,500,630,710,800,900,1000\}\,[mm]$. Time, speed and vibration frequency were modified in discreet manner, correspondingly: $\Delta t = 0.00005\,[s]$,

$\Delta V = 0.25\ [m/s]\ (V \le 2)$, $\Delta f = 0.25\ [1/s]\ (f \le 15)$. On the basis of Fig. 7 can be determined that the minimum acceleration may take place in the first phase of excitation only at low travel speeds, or with very small vibration frequencies not present in real structures. Also using $a_s$ factor does not enable to clearly separate the cases of the minimum in the first or second stage of excitement (Fig. 8).

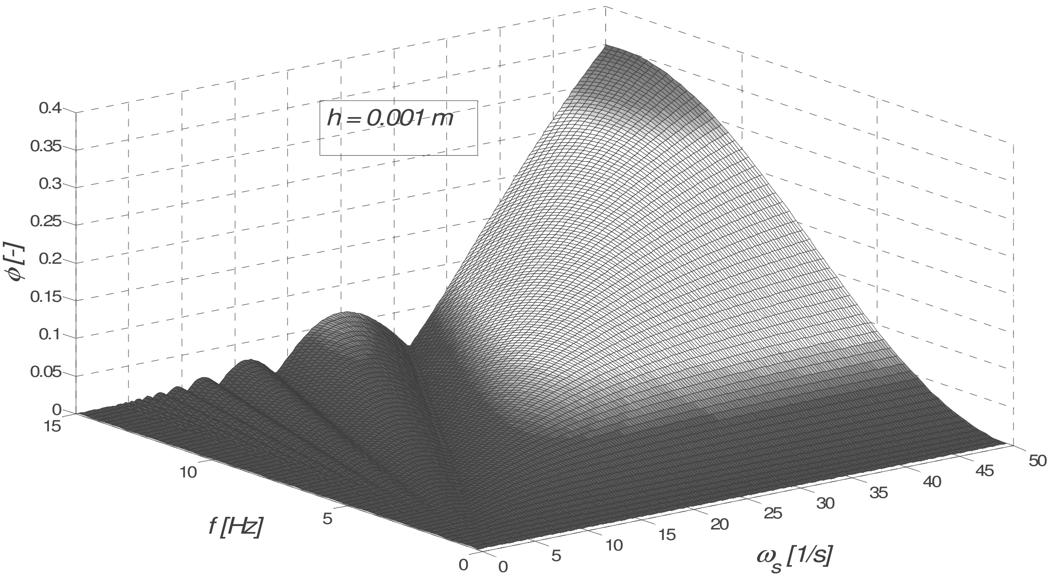


Fig. 5. The dynamic coefficient depending on the system's free vibration frequency and collision parameters
Rys. 5. Współczynnik dynamiczny w zależności od częstości drgań własnych ustroju i parametrów zderzenia

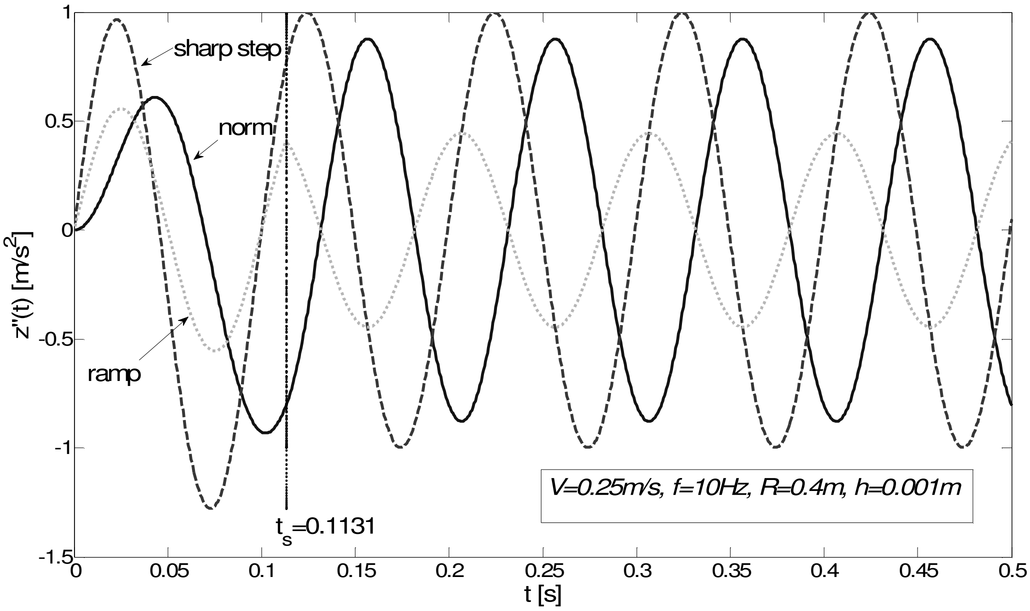


Fig. 6. Shape of acceleration function in time (minimum occurs at $t<t_s$)
Rys. 6. Przebieg funkcji przyśpieszenie w czasie (minimum występuje przy $t<t_s$)

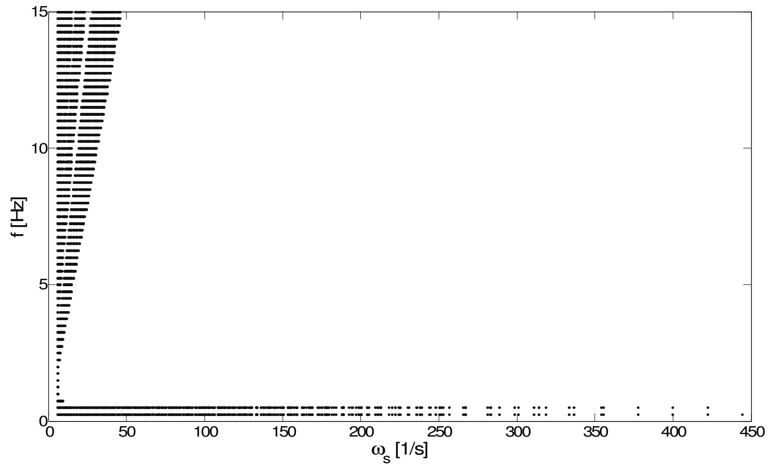


Fig. 7. Occurrence of the minimal acceleration values in the first phase of collision during kinematic excitation (points on the graph)

Rys. 7. Występowanie minimalnych wartości przyśpieszenia w pierwszej fazie podczas wymuszenia kinematycznego (punkty na wykresie)

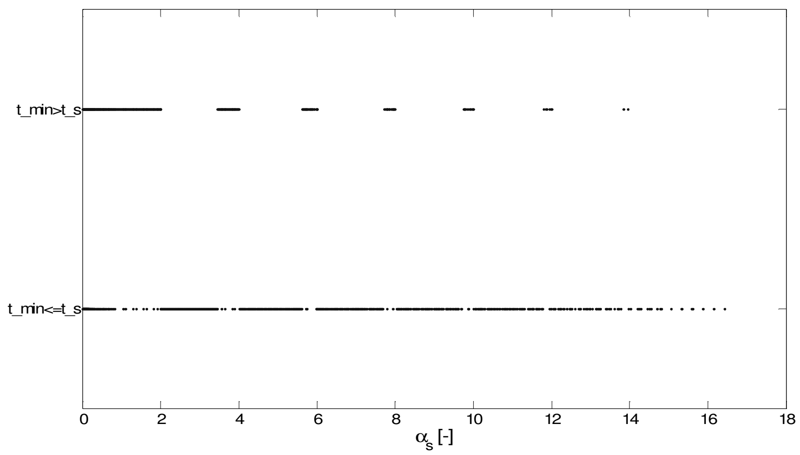


Fig. 8. Dependence of occurrence of minimal acceleration at the moment (points on the graph) to coefficient $\alpha_s$

Rys. 8. Zależność chwili wystąpienia minimum przyśpieszenia (punkty na wykresie) od współczynnika $\alpha_s$

Fig. 9 shows the dependence of the dynamic coefficient on free vibration frequency for the selected speeds: 0.3 m/s, 1 m/s and 2 m/s. Shown are three families of curves, each corresponding to a different height step. Similar families of curves are on the graph of dependencies of the dynamic coefficient on the speed for fixed free vibrations frequencies (Fig. 10).

## 4. SHARP OR ROUND STEP

Assuming completely rigid wheel collision with the step, it can be accepted that a wheel centre acts in uniform circular motion with the angular velocity $\omega_s' = \alpha / t_s = \alpha V / e$ (Fig. 2,3). In this case, the elevation function is:

$$h(t) = h + R\cos(\alpha - \omega_s' t) - R \quad t \le t_s; \quad h(t) = h \quad t > t_s. \tag{12}$$

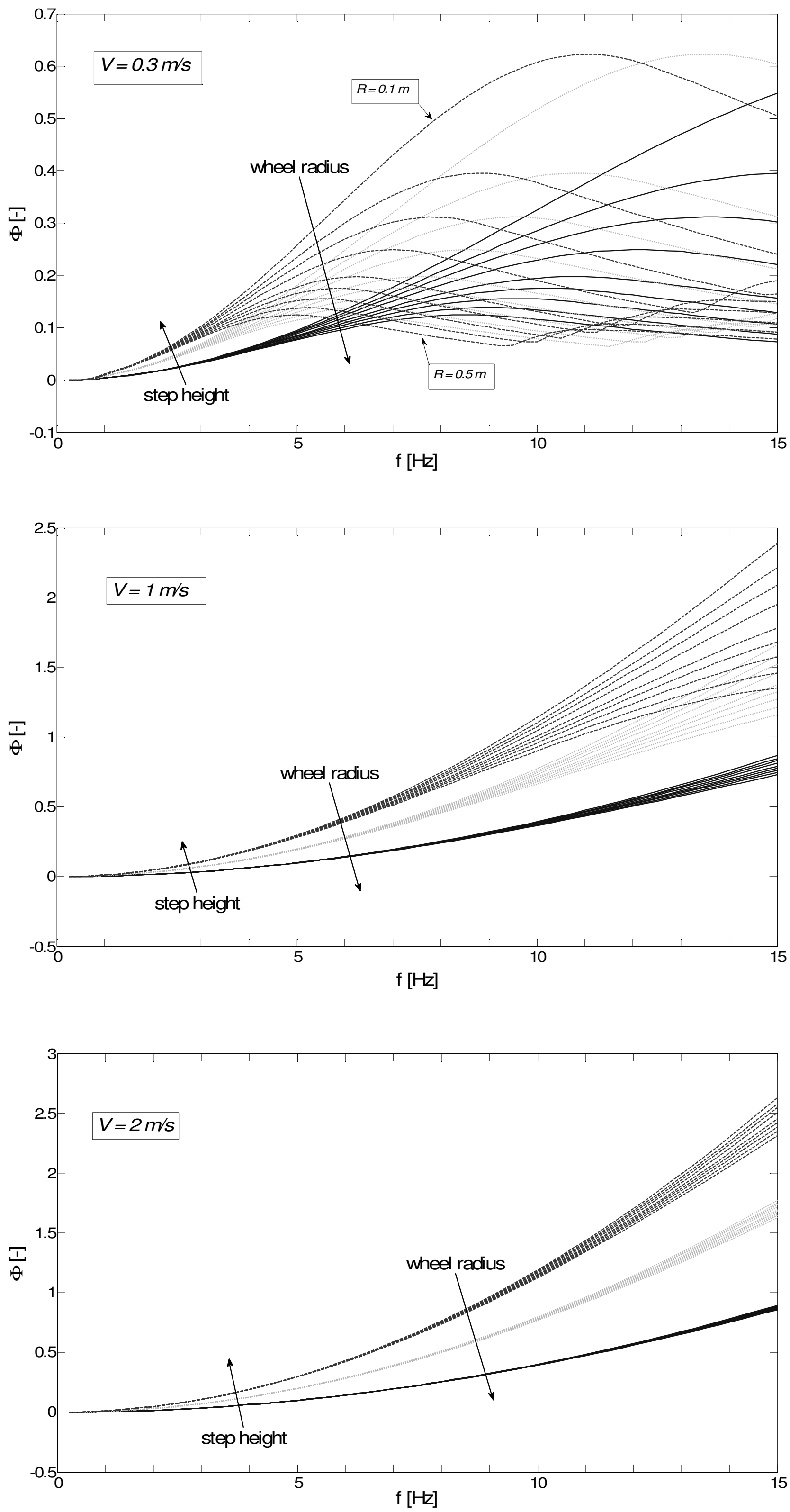


Fig. 9. Dynamic coefficient depending on free vibration frequency calculated according to the norm for chosen values of velocity

Rys. 9. Współczynnik dynamiczny obliczony według normy w zależności od częstości drgań własnych dla wybranych wartości prędkości

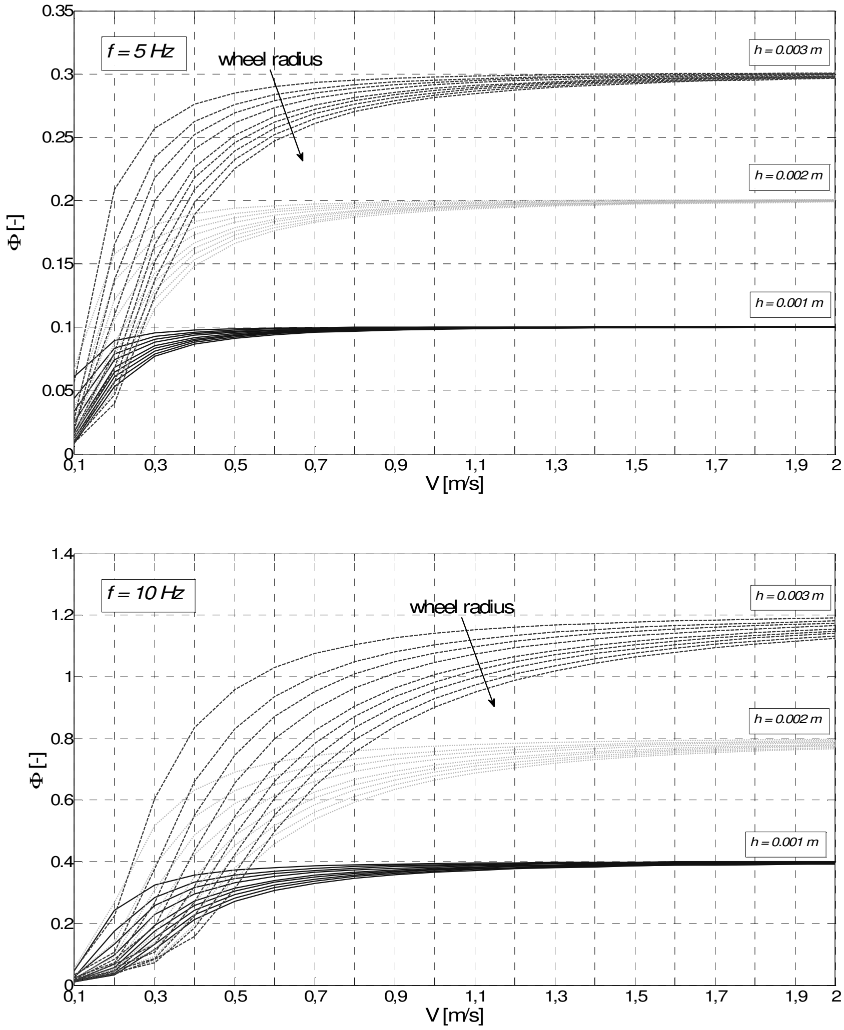


Fig. 10. Dynamic coefficient depending on travel velocity calculated according to the norm for chosen frequencies of free vibration

Rys. 10. Współczynnik dynamiczny obliczony według normy w zależności od prędkości jazdy dla wybranych częstości drgań własnych

Appropriate formulae for the displacement are given below. When $t \le t_s$:

$$z(t) = (R-h)(\cos\omega t - 1) + \frac{1}{2}\frac{\omega R}{\omega_s' + \omega}[\cos(\omega_s' t - \alpha) - \cos(\omega t + \alpha)]$$

$$-\frac{1}{2}\frac{\omega R}{\omega_s' - \omega}[\cos(\omega_s' t - \alpha) - \cos(\omega t - \alpha)] \qquad \omega_s' \ne \omega \tag{13}$$

$$z(t) = (R-h)(\cos\omega t - 1) - \frac{R}{2}\sin\alpha\sin\omega t + \frac{R}{2}\omega t\sin(\omega t + \alpha) \qquad \omega_s' = \omega$$

when $t>t_s$:

$$z(t) = (R-h)[\cos\omega t - \cos\omega(t-t_s)] + \frac{1}{2}\frac{R\omega}{\omega_s' + \omega}[\cos\omega(t-t_s) - \cos(\omega t + \alpha)]$$

$$-\frac{1}{2}\frac{R\omega}{\omega_s' - \omega}[\cos\omega(t-t_s) - \cos(\omega t - \alpha)] \qquad \omega_s' \neq \omega \tag{14}$$

$$z(t) = (R-h)\cos\omega t + h(1+\cos\omega t) + \frac{1}{2}\pi R\sin(\omega t + \alpha) \qquad \omega_s' = \omega$$

Acceleration can be determined in the same way as the previous, from formula (7). It has the same limit value: $\lim_{\omega_s' \to \infty, t_s \to 0} \ddot{z} = \omega^2 h \cos\omega t$, so $\lim_{\omega_s' \to \infty, t_s \to 0} \ddot{z}_{extr} = \pm\omega^2 h$.

The dependence of the dynamic coefficient on its free vibration frequency at a speed of 0.3 m/s is shown in Fig. 11. As you can see the results differ significantly from obtained according to the norm. In contrast to the norm, functions *Φ(f)* in the graph are increasing monotonically.

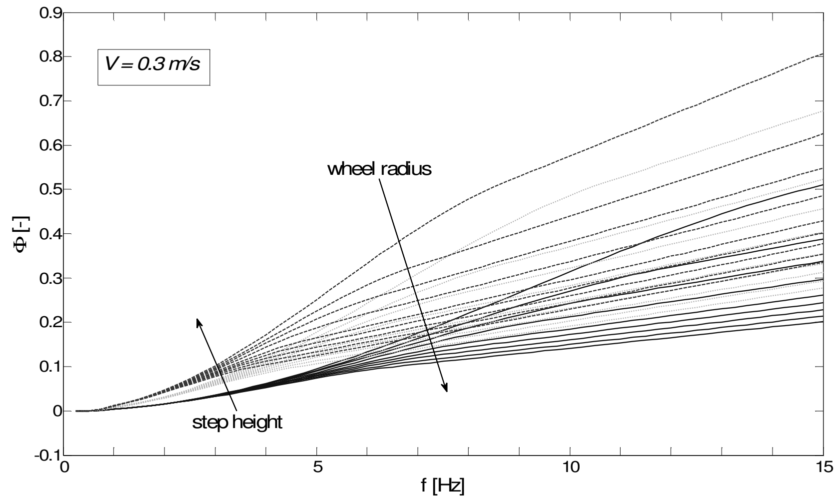


Fig. 11. Dynamic coefficient depending on free vibration frequency for 0.3 m/s speed calculated for sharp step
Rys. 11. Współczynnik dynamiczny obliczony dla ostrego progu w zależności od częstości drgań własnych przy prędkości 0,3 m/s

At low speeds the relative differences in values of coefficients $\Phi$ calculated for the sharp step and according to the norm ($\Delta\Phi=\Phi-\Phi_N$) exceed in extreme cases several hundred percent. With the increase of the speed differences are decreasing. Even at the speed of 1m/s within the frequency range from 2 to 10Hz differences are less than 5% (Fig. 12). It is noted that the larger are the step height and radius of wheel, the greater are differences.

## 5. RECTANGULAR QUASI PULSE OF ACCELERATION EXCITATION

The work [5] is considering rectangular quasi pulse of acceleration excitation as the approximation of a completely rigid collision. The results presented in [5] differ significantly from those obtained from formulae (13) and (14). Probably the reason is the adoption of zero initial conditions, which is equivalent to the action of excitation directly to the mass, rather than on the elastic bond.

From the formula (12) follows that, the wheel centre moves with centripetal acceleration $\ddot{h}(t) = -R\omega_s'^2\cos(\alpha - \omega_s' t)$. For small angles $\alpha$ is $\sin\alpha \approx \alpha$, $\cos(\alpha - \omega_s' t) \approx 1$. Therefore $\omega_s' = V/R$. Hence $\ddot{h}(t) \approx -V^2/R = -a$.

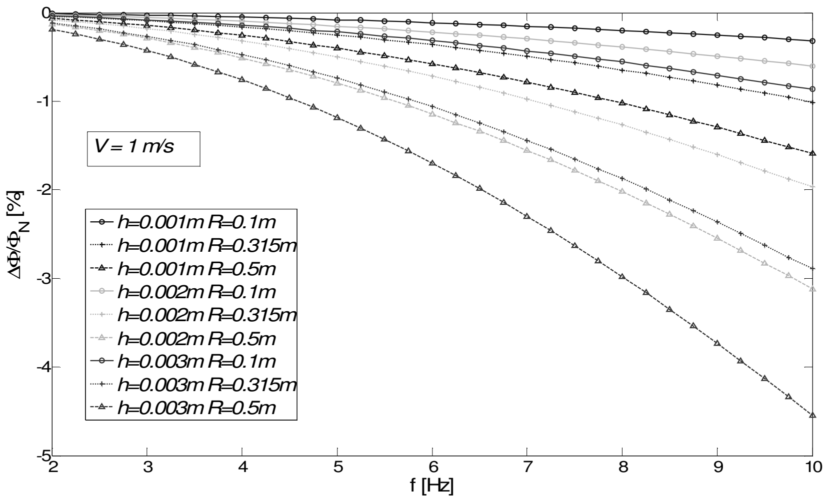


Fig. 12. Relative difference of dynamic coefficients (sharp step and norm) for 1 m/s speed
Rys. 12. Względna różnica współczynników dynamicznych (ostry próg i norma) przy prędkości 1 m/s

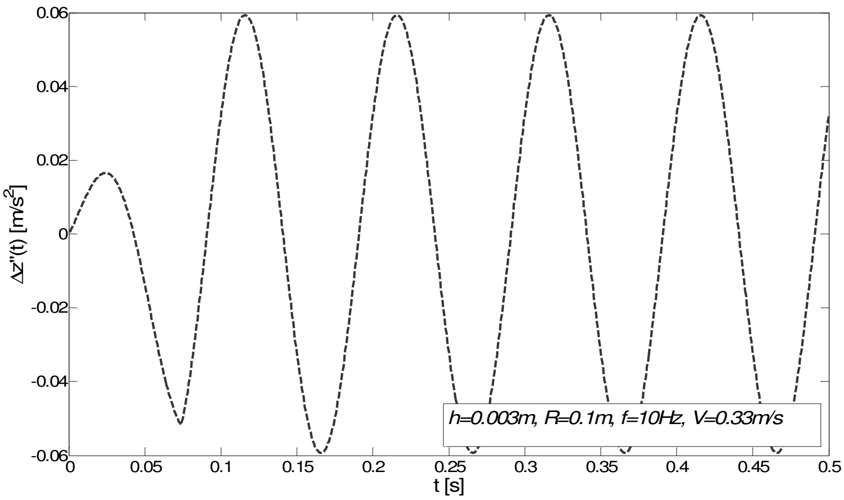


Fig. 13. Differences of acceleration function: sharp step and acceleration pulse
Rys. 13. Różnice funkcji przyśpieszenia: ostry próg i impuls przyśpieszenia

Differentiating twice (2), accepting $\dot{h}(t) = -a[H(t) - H(t-t_s)]$, where *H(t)* the Heaviside's function and substituting $w = \ddot{z}$ the equation with respect to acceleration is received:

$$\ddot{w} + \omega^2 w = -\omega^2 a[H(t) - H(t-t_s)] \tag{15}$$

Initial conditions (using (8)):

$$\begin{aligned} w(0) &= \ddot{z}(0) = \omega^2[h(0) - z(0)] = 0; \\ \dot{w}(0) &= \dddot{z}(0) = \omega^2[\dot{h}(0) - \dot{z}(0)] = \omega^2 \dot{h}(0) = \omega^2 \omega_s' R \sin\alpha \approx \omega^2 Ve / R. \end{aligned} \tag{16}$$

Hence the searching acceleration is:

$$\ddot{z} = \ddot{w} = \frac{\omega Ve}{R}\sin \omega t - a[1-\cos \omega t]H(t) + a[1-\cos \omega(t-t_s)]H(t-t_s) \tag{17}$$

Even for the greatest possible value of the angle differences between the results obtained from the formula (17) and the results for the case of a sharp step is negligible (Fig. 13).

## 6. THE RAMP EXCITATION

The case of pulse-step (ramp) excitation is also analyzed:

$$h(t) = \frac{h}{t_s}t \qquad t \le t_s\,; \quad h(t) = h \quad t > t_s \tag{18}$$

The following results are obtained:

$$\begin{aligned} z(t) &= \frac{h}{t_s}[t - \frac{1}{\omega}\sin \omega t] \qquad t \le t_s \\ z(t) &= \frac{h}{t_s}[t_s + \frac{1}{\omega}\sin \omega(t-t_s) - \frac{1}{\omega}\sin \omega t] \quad t > t_s \end{aligned} \tag{19}$$

Hence, the acceleration in the second phase of excitation is (after the necessary transformations):

$$\ddot{z}(t) = 2\frac{\omega h}{t_s}\sin\frac{\omega t_s}{2}\cos \omega(t - \frac{t_s}{2}) \tag{20}$$

A major drawback of excitation in the form (18) is the discontinuity of the derivative at the point $t=t_s$, which causes a discontinuity of adequate solutions. As with other forms of excitation with the increase in speed ($t_s \to 0$) acceleration amplitude tends to $\omega^2 h$ value.

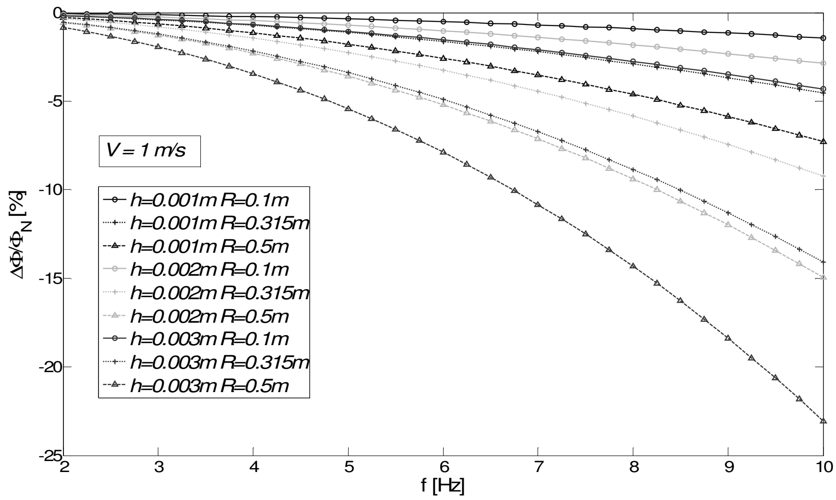


Fig. 14. Relative difference of dynamic coefficients (ramp and norm) for 1 m/s velocity
Rys. 14. Względna różnica współczynników dynamicznych (ramp i norma) przy prędkości 1 m/s

Like in the previous case relative differences in the values of the dynamic coefficients, calculated for the ramp excitation according to the norm, are decreasing with increasing speed. At a speed of 1m/s are still quite large (<25%, Fig. 14).

## 7. DISCUSSION OF RESULTS AND CONCLUSIONS

Each of these examined excitation functions are far from reality. Designation of an appropriate trajectory of centre of wheel during the travel of crane on roughness is a difficult task because of the nature of the phenomenon. It seems reasonable to consider the wheel travel in terms of impact. During the collision of the wheel with the rail local deformation will occur, which in compression phase will result in the wheel centre approaching the rail (decreasing radius), and then in the restitution phase the remoteness (radius increases). Component of velocity in the normal direction of impact may be so large that according to the literature (e.g. [6]) elasto-plastic collision will occur.

It can be assumed that the form of excitation function selected in the norms [8, 9] is a compromise between the ease of obtaining a mathematical solution, and the attempt to take into account the effects of a collision. In the absence of other possibilities should be taken to excitation (5) given in the norm and resulting from it formulae (bearing in mind the defects of the above-mentioned).

Analyses carried out in the work clearly confirm the trivial fact that the dynamic coefficient increases with step height and decreases with the increase of wheel radius. Rounding the step increases radius taken to the calculations. The dynamic coefficient also increases with increase of free vibrations frequency (except of cases with small travel velocity and the excitation according to the norm).

With the increase of travel velocity dynamic coefficient tends to the limit value. Form of kinematic excitation is increasingly less relevant. An important role is played by the step height and frequency of the system's free vibration.

The actual step height is taken into account in the presented results of the calculations, which influences the parameters of a collision. In the calculation of the dynamic coefficient, the possible height on which crane's mass centre can be elevated, is needed to be taken into account: $h_s=kh$, where $k$ – proportion coefficient, depending, inter alia, on the number of wheels [4]. The values given in the charts (ex. 9 and 10) should be multiplied by the estimated value of coefficient $k$ ($0<k<=1$).

## References

1. Markusik S.: *Wyznaczanie obciążeń konstrukcji stalowych dźwignic na podstawie norm europejskich.* Transport Przemysłowy 2/2001, p. 25-31.
2. Grabowski E., Malcher K.: *Zmiany w normowych zasadach ustalania obciążeń ustrojów nośnych dźwignic*. Transport Przemysłowy 2/2003, p. 24-31.
3. Grabowski E., Kosiara A.: *Wpływ sztywności kontaktowej kół jezdnych na drgania ustroju nośnego suwnicy.* Transport Przemysłowy 4/2004, p. 34-37.
4. Grabowski E., Kulig J.: *Wyznaczanie obciążeń dźwignic wywołanych jazdą po nierównościach według norm międzynarodowych*. Transport Przemysłowy 1/2007, p. 38-40.
5. Grabowski E., Kulig J.: *Metoda obliczania obciążeń dźwignic wywołanych jazdą po nierównościach.* Transport Przemysłowy 4/2007, p. 33-37.
6. Johnson K.L.: *Contact mechanics*. Cambridge University Press, 1985.
7. Norma PN-ISO 8686-1:1999. *Dźwignice. Zasady obliczania i kojarzenia obciążeń. Postanowienia ogólne.*
8. Norma PN-EN 13001-2:2004. *Bezpieczeństwo dźwignic. Ogólne zasady projektowania. Część 2: Obciążenia.*

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