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ANALYTICAL BASIS OF TECHNOLOGY ASYMMETRICAL ALLOCATION OF CARGO MASSES COMMON CENTRE IN WAGONS

Summary. The article gives the account of results of the analytical basis of technology asymmetrical cargo allocation in a wagon during rolling stock movement on track unevenness waves. The finite analytical formulas have been obtained for determination of response of wagon bogie spring sets in case of simultaneous displacement of cargo masses common centre longitudinally and sideways to the wagon, which make it possible to ground the cargo allocation according to admissible value of wagon vertical dynamic addition coefficient.

АНАЛИТИЧЕСКОЕ ОБОСНОВАНИЕ ТЕХНОЛОГИИ НЕСИММЕТРИЧНОГО РАЗМЕЩЕНИЯ ОБЩЕГО ЦЕНТРА МАСС ГРУЗОВ В ВАГОНАХ

Аннотация. В статье изложены результаты аналитическое обоснование технологии несимметричного размещения грузов в вагоне при движении подвижного состава по волнам неровности пути. Получены конечные аналитические формулы для определения реакции комплектов пружин тележек вагона при одновременном смещении общего центра масс груза как вдоль, так и поперёк вагона, которые позволяют обосновать технологию размещения грузов по допустимому значению коэффициента вертикальной динамической добавки вагона.

1. FORMULATION OF A PROBLEM

According to Appendix 14, Agreement on International Freight Rail Transportation [1], as a rule, all cargoes on the open rolling stock (ORS) should be allocated using the first technology (symmetric allocation of cargo masses common centre MC relative to wagon symmetry axes). When this technology is unfeasible for some reason, i.e. cargo geometrical parameters as well as cargo allocation and fastening conditions make it impossible to fall the cargo within loading gauge, other cargo allocation technologies are applied. However, the existing ORS cargo allocation technologies are still analytically unjustified.

Assuming a carriage underframe to be a perfect rigid body (i.e. straight), and disregarding its rotation due to spring set deformation, it is essential to ascertain limits for cargo common gravity decentration along [xM] (towards the leading bogie) as well as across [yM] (towards the external rail thread) of the carriage by the criterion of deflection of bogie spring set equivalent forces not more than 25% of the mean value of the “cargo – carriage underframe –bolsters” mechanical system, or admissible carriage vertical dynamic addition coefficient value k_{ad} [2].

1.1. Man-made assumption

O is assumed to be a home, consilient with the axes transection centre of carriage symmetry, i.e. in the carriage floor centre. Ox - axis will go lengthwise to the wagon, Oy - across the wagon, Oz - as shown in Fig.1.

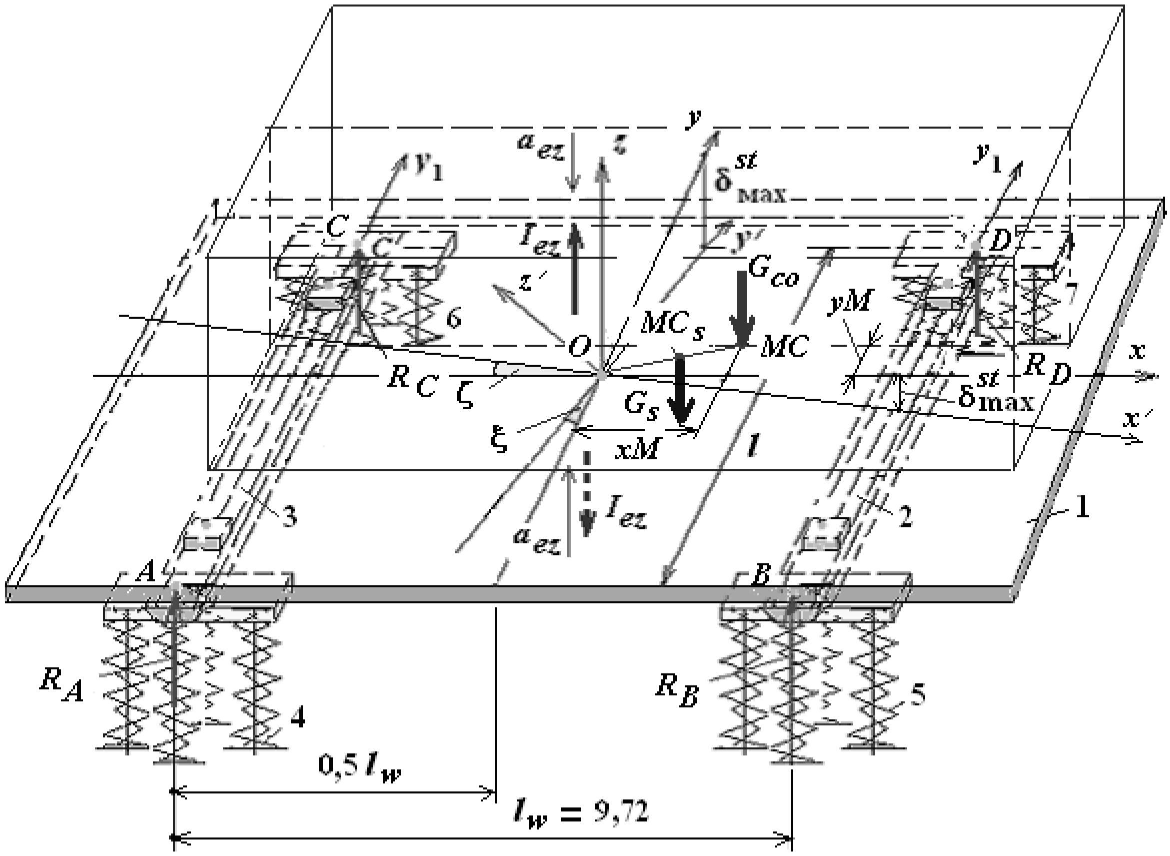

Fig. 1. Determination of a bogie spring set response: 1 – carriage underframe, 2 и 3 – bolsters , 4 – 7 – spring sets

Рис. 1. Определение реакции комплектов пружин тележки: 1 – рама вагона, 2 и 3 – надрессорные балки, 4 – 7 – комплекты пружин

Fig. 1 shows: G_{co} - overall cargo weight (for piece loads the cargo weight is G); $G_c = Q_s$ - “cargo – carriage underframe – bolster” mechanical system gravity; $\overline{a}_{ez}$ - vertical acceleration of transportation of any “cargo – carriage underframe – bolster” mechanical system point; I_{ez} - vertical inertia force of transportation; xM and yM - cargo common masses centre displacement MC relative to transverse and longitudinal wagon symmetry axes.

We assume that all points of “cargo – carriage underframe – bolster” mechanical system are moving by vertical acceleration of transportation $\overline{a}_{ez}$, emerging due to track unevenness waves.

We shall describe the origin of vertical inertia force of transportation [5]. The loaded wagon acceleration of transportation along the vertical axis $\overline{a}_{ez}$ is explained by resilient elements (spring sets) between lateral underframes and bolsters and the condition of the track, on whose unevenness waves the train is moving. That's why the vertical force of transportation $\bar{I}_{ez}$ is formed and perceived by fastening resilient elements.

Therefore, acceleration of transportation of a loaded wagon along the vertical axis will simulate " a loaded wagon bouncing" phenomenon in mathematical model due to rolling-stock movement on track unevenness waves, the height of the wave depending on proper maintenance of the track maintenance standards. Furthermore interaction of "track – wagon – cargo –fastening" mechanical system in vertical direction will be taken into account if any deviation from track maintenance standards takes place.

When calculating indeterminate external constraint reactions, vertical inertia force of transportation $\bar{I}_{ez}$ for loaded wagons is estimated in accordance with maximum standardized value of acceleration along the vertical axis $\bar{a}_z = \bar{a}_{ez}$ [2] which results from movement of a wagon at a station-to-station block $a_{ez} = (0{,}46 - 0{,}66)g$. Acceleration value may substantially exceed accepted value if track maintenance standards are not properly followed.

The inertia force of transportation is an active force inasmuch as its impact on a rested object can set it in motion.

$\bar{I}_{ez}$ force is conditionally applied to C cargo masses center (not shown in Fig. 1) and its effect is experienced by external constraints such as railway wagon floor, flexible elastic (tension wire and framing) and thrust (blocking lumbers) or control safety devices for frieght transportation.

When determinating the direction of inertia force of transportation, unfavourable incidents should be considered. If we take into account the direction of inertia force, then unfavourable incident for the spring set of a bogie is the case when vertical acceleration of transportation $\bar{a}_{ez}$ is directed upwards [4]. In this case the spring sets disposed along decentration of cargo masses will be overloaded with additional vertical inertia force of transportation $\bar{I}_{ez}$. Inertia force of transportation $\bar{I}_{ez}$ in analytical model is directed from an object.

We assume that a cargo common masses center MC has coordinates $x_M = xM$ (towards a front bogie) and $y_M = yM$ (towards an external rail thread) which are equal to cargo displacement lengthwise as well as sideways of the wagon. Let us determine elastic forces of bogie spring sets $\bar{F}_i$ in the form of constraint reaction R_A, R_B, R_C and R_D for the case when common masses center of «cargo – carriage underframe – bolster» system has coordinates $x_C = xC$, $y_C = yC$ (In Fig. 1 notshown) and a cargo common masses center MC is displaced relative to transverse and longitudinal wagon symmetry axes by values xM and yM .

The mass centre of the mechanical system "cargo – carriage underframe – bolster" MC_s, according to the theorem on the resultant of plane forces (Varinyon's theorem) under specified values of the cargo allocation (displacement) lengthwise the wagon (towards the front bogie) by the value $x_M = xM$ and across the wagon (towards the external rail thread) $y_M = yM$, is defined by the the following formulas

$$x_c = xC = xM \frac{G_{co}}{Q_s}, \tag{1}$$

$$y_c = yC = yM \frac{G_{co}}{Q_s}, \tag{2}$$

where G_{co} - overall cargo weight (for piece loads the cargo weight is - G), kN;

Q_s - gravity (pressure) of the mechanical system „cargo – carriage underframe –bolster) taking into consideration the carriage movement acceleration on track unevenness waves (i.e. in dynamics)

$$Q_s = Q_{st} + (I_{ez} + I_{ezw} + 2I_{ezcu}), \quad (3)$$

where Q_{st} - static load on bogie spring sets from impact of the gravity of the mechanical system „cargo – carriage underframe – bolster", kN

$$Q_{st} = G_w + 2G_{cu} + G_{co}; \quad (4)$$

I_{ezw} and I_{ezcu} - vertical forces of inertia forces of transportation of the carriage underframe and bolster respectively [5].

The expression (3) taking into account formulas (4) will be finally presented as

$$Q_s = k_d Q_{st}, \quad (5)$$

where k_d - wagon vertical dynamics coefficient (or quasi-static coefficient)

$$k_d = 1 + k_{ad} \quad (6)$$

taking into consideration that k_{ad} is the wagon vertical dynamic supplement coefficient (or allotment of vertical acceleration of transportation in relation to g)

$$k_d = \frac{a_{ez}}{g}. \quad (7)$$

Further we shall bear in mind that the response of each bogie spring set equals

$$R_{st} = \frac{Q_{st}}{4}, \quad (8)$$

where 4 is the total amount of spring sets in carriage bogies.

We shall consider asymmetric allocation of piece loads relative to longitudinal and transverse wagon symmetry axes, for example, towards bearings B and D, by values xM and yM. Position of the cargo common masses centre MC relative to transverse xM and longitudinal yM wagon symmetry axes (m) is taken depending upon the cargo weight and height of the loaded wagon common masses centre above the rail level according to Table 9 and 10, Appendix 14 to International Freight Rail Transportation Agreement (IFTA) .For instance, for cargo weight G = 294,3 kN and height of the loaded wagon common masses centre above the rail level 2,3 m - xM = 1 440 and yM = 280 mm.

At the same time, due to the masses centre MC_s displacement of the mechanical system" cargo – carriage underframe" , the wagon fore and rear bogie underframe and bolsters will be sloping towards the bearing by angles ξ и ζ, overloading spring sets of bearings A and B (or C and D), and relieving the similar springs of bearings B and D (or A and C). The train loaded wagon will have the same inclined position of the loaded underframe and bolsters (Fig. 2).

The angle of slope of the loaded wagon underframe from MC displacement relative to transverse and longitudinal symmetry axes are defined by formulas

$$\xi = arctg\left(\frac{\delta_{zl}}{xM}\right); \quad \zeta = arctg\left(\frac{\delta_{zb}}{yM}\right), \tag{9}$$

where δ_{zl} and δ_{zb} – static sagging of spring sets depending on overall cargo weight G_{co}, applied along longitudinal and transverse axes and determined by experiment, mm (we shall bear in mind that admissible δ_{st} = 46 – 50 mm).

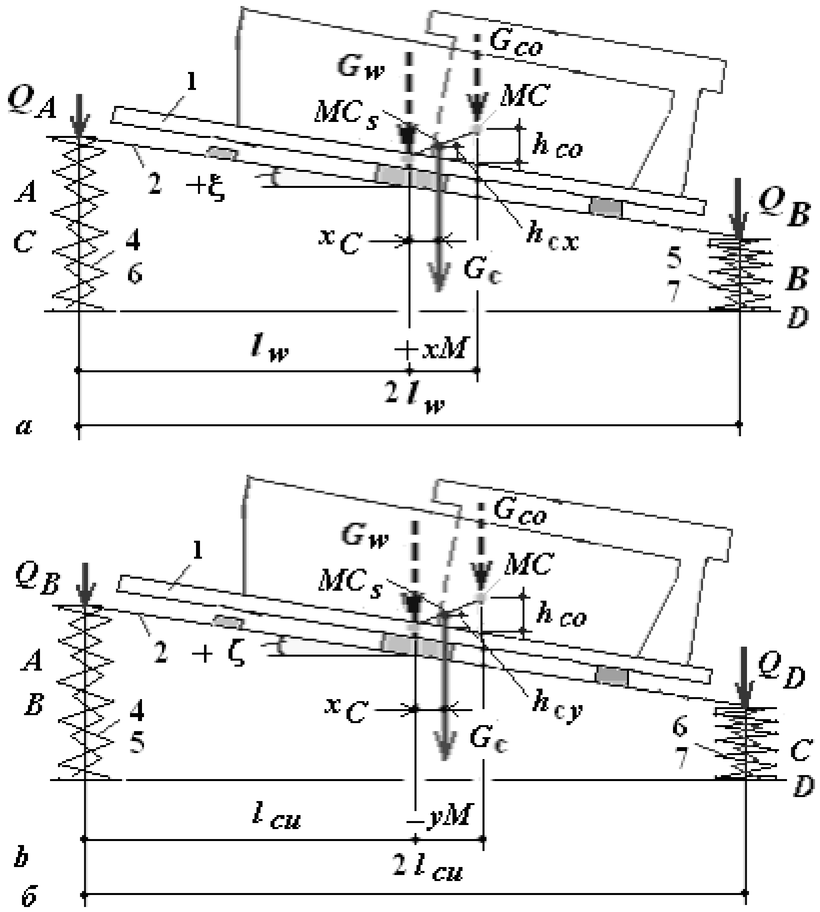

Fig. 2. Redistribution of carriage underframe pressure force on bogie spring sets: *a* – in cargo displacement lengthwise the wagon; *b* – in cargo displacement crosswise the wagon

Рис. 2. Перераспределение силы давления рамы вагона на комплекты пружин тележек: *а* – при сдвиге груза вдоль вагона; *б* – при сдвиге груза поперёк вагона

The loaded platform underframe through bearers and center bowls rests on 2 fore and 3 bogie bolsters. We assume that gravity of the cargo, carriage underframe and bolsters including the vertical inertia force of transportation $\bar{I}_{ez}$ is the force applied from the mechanical system "cargo – carriage underframe – bolster" on bogie spring sets as Q_s.

In this case, pressure force in Q_s will have an effect on bogie spring sets, its point of application being determined by masses centre coordinates $x_C = xC$ and $y_C = yC$, resulting in 4 – 7 bogie spring sets response as R_A and R_B, R_C and R_D. These constraint forces are slightly different from Q_s/4.

2. METHODS OF SOLUTION

We shall make use of the theorem on the resultant of plane forces (Varinyon’s theorem) and the notion of the center of gravity of mechanical system «cargo – carriage underframe – bolster», as wagon stability is determined by the position of the gravity center of MC_s system but not of MC of the cargo [3, 4] . This consideration of the masses centre of «cargo – carriage underframe – bolster» mechanical system is more general [3] since wagon stability is determined by position of the system of the masses centre MC_s but not MC of the cargo.

3. SOLUTION RESULTS

We shall consider conditions for an equilibrium of parallel forces space system

$$\sum_{k=1}^{n} F_{kz} = 0; \qquad R_A + R_B + R_C + R_D - Q_s \cos\xi \cos\zeta = 0; \tag{10}$$

$$\sum_{k=1}^{n} m_x(\overline{F}_k) = 0; \ (R_C + R_D)l - (R_A + R_B)l - Q_s \cos\zeta y_c - Q_s \sin\zeta h_{cy} = 0; \tag{11}$$

$$\sum_{k=1}^{n} m_y(\overline{F}_k) = 0; \ (R_A + R_C)l_w - (R_B + R_D)l_w + Q_s \cos\xi x_c + Q_s \sin\xi h_{cx} = 0; \tag{12}$$

where $l = l_{cu}$ - is half the length of the bolster, m (2l_{cu} =2,036 m); l_w - is half the wagon base m (2l_w = 9,72 m);

h_{cx} and h_{cy} - is the height of application of the masses center (MC_s) system along the longitudinal and transverse wagon axisrespectively, m

$$h_{cx} = h_{co}\frac{x_c}{xM}; \quad h_{cy} = h_{co}\frac{y_c}{yM} \tag{13}$$

$$h_{cx} = h_{co}\frac{x_c}{xM}; \quad h_{cy} = h_{co}\frac{y_c}{yM} \tag{13a}$$

Taking into consideration that h_{co} is the height of application of the cargo common masses center (MC_s) above the wagon floor, which is selected according to the overall cargo weight (G_{co} = G) and MC_s above rail level. (Fig. 2.2a,b)

$$h_{co} = \frac{H_{co}(G+G_t)-G_t h_t}{G}. \quad (14)$$

Here H_{co} - is the height of MC_s above rail level, its acceptable values depend on the overall cargo wieght G_{co} = G) and are taken according to Table 10, Appendix 14, Agreement on International Freight Rail Transportation. For instance, if $G_{co} \le 100$ kN, then H_{co} s taken 1,2; 1,5 и 2 m; if 300 $\le G_{co} \le 550$ kN, then H_{co} – 1,2; 1,5; 2 и 2,3 m; if 550 $\le G_{co} \le 670$ kN, then H_{co} – 1,5; 2 & 2,3 m; if $G_{co} \ge 670$ кN, then $H_{co} \le$ 2,3 m.

The system made up of 3 formulas contains 4 inderterminates. The task is statically inderterminant.

In order to solve this problem, regarding carriage underframe as perfectly rigid body, we determine displacement of the center of the carriage underframe, which can be expressed in terms of deformation (excessive bends) δ_i (in mm) of diagonally opposite bogie spring sets. Under that assumption, recording condition of deformation compatibility of bogie spring sets and making use of a physical equation, which ties together force and displacements (Hooke's law),we shall have an additional equation of equilibrium 4 (plus three already made equations of equilibrium (10) – (12)

$$R_A + R_D = R_B + R_C. \quad (15)$$

Thus, in order to determine the bogie spring sets response 4-7 in the form of R_A and R_B, R_C and R_D we have the following system of 4 linear algebraic equations.

$$\begin{gathered} R_A + R_B + R_C + R_D = Q_s \cos\xi \cos\zeta; \\ (R_C + R_D)l - (R_A + R_B)l = Q_s \cos\zeta y_c + Q_s \sin\zeta h_{cy}; \\ (R_A + R_C)l_{\text{в}} - (R_B + R_D)l_{\text{в}} = -Q_s \cos\xi x_{\text{c}} - Q_s \sin\xi h_{\text{cx}}, \\ R_A - R_B - R_C + R_D = 0. \end{gathered} \quad (16)$$

The solution of the system of linear algebraic equations (16) for determination of reaction of bogie spring sets R_A and R_B, R_C and R_D by analytical (symbolic) method in MathCAD computing environment [6], after elementary mathematical calculations we shall have it in the following form

$$\mathrm{RA} := \frac{\mathrm{Qs}}{4}\cdot\left(\cos\xi\cdot\cos\zeta - \frac{\cos\xi\cdot\mathrm{xc} + \sin\zeta\cdot\mathrm{hcx}}{\mathrm{lw}} - \frac{\cos\zeta\cdot\mathrm{yc} + \sin\zeta\cdot\mathrm{hcy}}{1}\right); \quad (17)$$

$$\mathrm{RB} := \frac{\mathrm{Qs}}{4}\cdot\left(\cos\xi\cdot\cos\zeta + \frac{\cos\xi\cdot\mathrm{xc} + \sin\zeta\cdot\mathrm{hcx}}{\mathrm{lw}} - \frac{\cos\zeta\cdot\mathrm{yc} + \sin\zeta\cdot\mathrm{hcy}}{1}\right); \quad (18)$$

$$\mathrm{RC} := \frac{\mathrm{Qs}}{4}\cdot\left(\cos\xi\cdot\cos\zeta - \frac{\cos\xi\cdot\mathrm{xc} + \sin\zeta\cdot\mathrm{hcx}}{\mathrm{lw}} + \frac{\cos\zeta\cdot\mathrm{yc} + \sin\zeta\cdot\mathrm{hcy}}{1}\right); \quad (19)$$

$$\mathrm{RD} := \frac{\mathrm{Qs}}{4}\cdot\left(\cos\xi\cdot\cos\zeta + \frac{\cos\xi\cdot\mathrm{xc} + \sin\zeta\cdot\mathrm{hcx}}{\mathrm{lw}} + \frac{\cos\zeta\cdot\mathrm{yc} + \sin\zeta\cdot\mathrm{hcy}}{1}\right). \quad (20)$$

Analysis of the obtained results. Therefore, it is evident that if values of cargo masses center displacement are positive lengthwise as well as sideways the wagon $x_c = xC$ and $y_c = yC$ (this is equivalent to cargo xM diplacement lengthwise as well as sideways the wagon yM) reactions of bogie spring sets R_A and R_C, positioned opposite to cargo displacement wagon side are minimum, and those positioned on the side of cargo displacement R_B and R_D are maximum. This is true to reality as spring sets B and D are overloaded and A and C are unloaded. Reactions R_A and R_C diverge from the average value Q_s /4 less, and quite the contrary - as for R_B and R_D.

Transverse stability of a loaded wagon according to technical specification of load allocation and fastening in rail wagons requires strict limitations, depending on cargo weight G and its masses center height H_{MC} relative to the rail level. Therefore, requirements for divergencies of reactions R_A and R_B, R_C and R_D not to exceed an average value of 25% or by acceptable value of coefficient of vertical dynamic supplement of the wagon k_{ad}, make it possible to find out utmost values for cargo displacement lengthwise [xM] as well as sideways of the wagon [yM] depending on its weight, using the resulting formulas $xM = f(G)$ and $yM = f(G)$ if H_{MC} is a fixed value.

We shall determine wagon vertical dynamic supplement coefficient k_{ad} of spring sets A (or C) and B (or D) using the following formulas

$$k_{adA} = \frac{R_A - R_{st}}{R_{st}}; \quad k_{adB} = \frac{R_B - R_{st}}{R_{st}}, \tag{21}$$

where R_{st} - is a static load applied to bogie spring sets when there are no vertical inertia forces of transportation (that is, when $a_z = 0$) (see formula (8)).

Calculation: The initial data, if there is a displacement of cargo lengthwise as well as sideways of the wagon of xM and yM values, are as follows: cargo weight G = 294,3 kN; weight of the transport container G_t = 215,82 kN; the height of the masses center of an empty wagon h_t = 0,8 m; the height of the cargo common masses center above the rail level (Table 9, Appendix 14 to Agreement on International Freight Rail Transportation) H_{co} = 1,2 m; the height of the cargo masses center above the wagon floor (formula (14)) h_{co} = 1,493 m; the weight of the carriage underframe G_w = 117,72 kN; the weight of the bolster G_{cu} = 4,415 kN.

Kinematic disturbance transmitted to a cargo from the track on a wagon a_{ez} = 0,46 g = 4,513 m/s^2 – acceleration of transportation along the vertical axis.

Inertia vertical forces of transportation, kN, of the cargo I_{ez} = 135,378, carriage uderframe I_{ezv} = 54,15, bolster I_{ezcu} = 12,03.

Static pressure on the bogie spring sets from “track – bolster – carriage underframe – cargo” mechanical system - Q_{st} = 420,85 kN (formula (4)). Static reaction of a single spring set - R_{st} = 105,21 kN (formula (8)). Coefficient of the vertical dynamic supplement of the wagon - k_{ad} = 0,46 (formula (7)).

Dynamic pressure on bogie spring sets from “track – bolster – carriage underframe – cargo” mechanical system - Q_s = 153,61 kN (formula (5)).

Displacement of the masses center of the system lengthwise as well as sideways of the wagon - x_c = 0,69 and y_c = 0,134 m ((1) and (2) formulas) if xM = 1,44 и yM = 0,284 m for the cargo weight G = 294,3 kN. The height level of the masses center of the system - h_{cx} = 0,715 and h_{cy} = 0,715 m (formulas (13)). The slope angle of the carriage underframe from displacement of cargo masses center lengthwise as well as sideways of the wagon ξ = 0,003 and ζ = 0,014 rad. (formula (9)).

As a result of the calculations performed we have the following data. If values of displacement of cargo weight G = 294,3 kN are positive lengthwise as well as sideways of the wagon, then reactions of bogie spring sets which are allocated on the opposite to cargo displacement part of the wagon are minimum (R_A = 109,69 и R_C = 153,25 kN) and those allocated on the displacement of cargo part – are maximum (R_B = 153,94 и R_D = 197,5 kN), i.e. spring sets B and D are overloaded and A and C are unloaded.

In case if $\xi = 0$ and $\zeta = 0$, reactions of bogie spring sets allocated on the opposite to displacement of cargo part of the wagon are minimum (R_A = 111,57 and R_C = 152 kN) and those allocated on the cargo displacement part are maximum (R_B = 155,17 и R_D = 195,65 kN).

The difference between reactions of spring sets equals 2% at a rough estimate, if we take into account a slope of the carriage underframe and allocation level of the cargo common masses center relative to the wagon floor - h_{co} (i.e. if $\xi \neq 0$ и $\zeta \neq 0$) and without considering them (i.e. if $\xi = 0$ и $\zeta = 0$). The results of the performed calculations indicate that if 1,2 $\leq h_{co} \leq$ 2,3 m then the difference between reactions of spring sets equals 2% - 6%.

In this context from now onwards we shall conduct calculations on normalization of cargo displacements lengthwise as well as sideways of the wagon without taking into account a slope of the carriage underframe and allocation level of the cargo common masses center relative to the wagon floor, i.e. $\xi = 0$ и $\zeta = 0$.

Coefficient value of the vertical dynamic supplement of the wagon ranges within 0,04 < k_{ad} < 0,877 if a minimum acceptable value of vertical acceleration of transportation a_{ez} = 0,46g. In case if $\xi = 0$ and $\zeta = 0$ the coeffecient value – 0,06 < k_{ad} < 0,86 that does not correspond to its minimum acceptable value [k_{ad}] = 0,3 – 0,6. Furthermore, safety transportation conditions may be endangered, undamaged condition of cargo not ensured and details and element parts of the rolling-stock can be damaged.

Hence, an important practical conclusion can be made that if displacement of the cargo common masses center is simultaneous lengthwise as well as sideways of the wagon as illustrated in Table 9 and 10 (Appendix 14 to the Agreement on International Freight Rail Transportation) MC_c displacement restrictions cannot be used as acceptable.

4. SUMMARY

1. On the basis of the conducted analytical research previously unknown in the theory of cargo allocation, analytic formulas have been derived to determine reaction of bogie spring sets if common masses center is allocated asymmetrically lengthwise as well as sideways of the wagon simulteneously, depending on forces applied to the bogie spring sets from “cargo – carriage underframe – bolsters” sys-

tem, taking into account vertical inertia force of transportation and cargo displacement lengthwise as well as sideways of the wagon. Generalized analytical formulas for determination of reaction of bogie spring sets allow us to evaluate their load carrying capacity for a particular case when the cargo common masses center is displaced along or across the wagon.

2. Cross stability of a wagon with cargo according to technical specification of cargo allocation and fastening in wagons depending on an average value of the force applied to bogie spring sets of “cargo – carriage underframe – bolsters” system including the vertical inertia force of transportation and height of its gravity center above the rail level requires more tight restrictions. For this reason in order to have the coefficient of a wagon vertical dynamic supplement within the acceptable limits $0{,}3 \le k_{ad} \le 0{,}6$, applying deduced analytical formulas, it is possible to ascertain extreme values of cargo displacement along as well as across the wagon in accordance with its weight taking into consideration acceleration of wagon movement if height of its gravity center relative to rail level is a fixed value.

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