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MATHEMATICAL MODEL OF EVALUATION OF THE EFFICIENCY OF INVESTMENT INTO ROADS

Summary. When implementing projects on motor roads, financial and economic analysis of specific projects are usually provided. This article deals with the mathematical model which enables to analyze the entirety of projects on the motor road network with the view of determining which of them would give benefit to society the fastest and could be given priority in case of implementation. By applying this model and using the LP search (abbreviation from Russian Linear search) multi-criteria optimizing method, it is possible to calculate the project payoff time and an expected profit through the reduction of public costs. Also, an example on realization of the model is provided with calculation of the benefit that society receives if gravel road is paved.

МАТЕМАТИЧЕСКАЯ МОДЕЛЬ ДЛЯ ОПРЕДЕЛЕНИЯ ЭФФЕКТИВНОСТИ ИНВЕСТИЦИЙ В АВТОДОРОГИ

Резюме. Внедряя дорожные проекты всегда сталкиваемся с их финансовым и экономическим анализом. В статье предлагается математическая модель, дающая возможность анализировать финансовые составляющие проектов, чтобы установить который из них окупится в кратчайший срок и естественно сможет занять приоритетное место при его внедрении. При реализации модели применен метод многокритеральной оптимизации „ЛП поиск“. Показан пример реализации модели, дающий возможность установить общественную пользу, асфальтируя гравийнные дороги.

1. INTRODUCTION

Until the 2nd half of the last century, a technical attitude towards the planning of motor roads prevailed. Road workers had to ensure the road permeability, design driving speed, and did not pay attention to the needs of road users.

Later the attitude changed. The operational costs of transport vehicles, the value of travel time, and losses associated with traffic accident or environment pollution were taken into assessment. The HDM-4 (The Highway Development and Management System) model [9] is used for calculating the expediency of investment into the motor roads in the world practice. The expediency of investments into the motor roads is calculated using the model, whereas the benefit to society provided by the project may be also calculated using a separate sub-model. Using this model a number of precise technical parameters have to be evaluated, as a result whereof sufficiently accurate calculations are obtained, yet this is not always convenient seeking to quickly compare the benefit of particular projects at one time.

The investigation into the condition of gravel roads of Lithuania and the justification of renovations are examined by Skrinskas [7]. A mathematical model for the assessment of economic expediency of renovation of the Lithuanian gravel roads KAMIS (the road asphalting model) is provided in the thesis on which basis the renovation costs and savings are calculated with consideration to the technical indicators of gravel roads, evenness of the road surface and the traffic indicator values. However, this model is rather used for assessing the technical indicators and it is applicable only to the assessment of the road paving projects.

This article is intended to provide a mathematical model on which analysis would enable to determine the period of return on investment according to the project costs and the reduction of public costs, by applying a multicriterial optimization (LP search).

To this effect it is necessary to determine the main parameters associated with costs (road operation, maintenance, accident rate, ecology); to make functional correlations between the main road parameters and the change in pecuniary costs; to make a generalized mathematical model for minimizing the public costs during the period selected.

2. ROAD USER COSTS (*RUC*) AND BENEFIT OF THE PROJECT

Implemented projects on motor roads, if this is not the construction of toll roads, practically do not bring any direct income. Irrespective of that, great sums of public funds collected from the taxpayers are allocated to investments into motor roads. It means that investments into motor roads are worthwhile, since they give not only social benefit, but also make premises for a more efficient use of vehicles (the speed, operational reliability increase, fuel costs reduce, etc.).

The components of RUC and persons residing in the road impact zone are as follows [3, 5, 9]:

- *Vehicle Operating Costs (VOC).* While driving on the road different costs are sustained, consisting of fuel, lubricants, maintenance, tyre costs, drivers' salary and other expenses. A part of these costs depends neither on the road nor on its condition, yet do other parts depend on irregularity of the road surface [4], number and width of traffic lanes, driving speed, road curviness and longitudinal gradient.
- *Annual Accident Costs (AC)* makes a great effect to society. The accident rate in Lithuania is one of the highest in Europe. An accident rate is an important indicator of economy, i.e. the less number of people die or are injured, or the number of transport vehicles are damaged, the greater economy is received. The price of accidents with deaths, injuries, and of the technical traffic accident is differentiated.
- *Travel Time Costs (TTC).* The travel time is expressed by generalized costs that combine travel time and pecuniary expenses. The time spent on travel is considered as costs with respect to an individual since he may not engage in any other activity at that time. The value of the time saved depends on the fact what an individual may create during that time. The costs of working time, non-working time and transportation of goods are differentiated. The length of travel depends on the length of the route chosen, transport vehicle, road conditions, permissible speed, technical characteristics of transport vehicle, traffic volume and traffic capacity.
- *Social and Environmental Effects (SEE).* The quantities of pollutants in case if the project is implemented and is not implemented are determined. The pollution of air, dust on gravel roads, noise, and hothouse effect should be analyzed.
- *Repair and Maintenance Costs (RMC).* The repair and maintenance costs consist of the direct costs of planning, designing and implementation of the structure, and operational (maintenance) costs (while implementing the project) of the structure (the costs of permanent maintenance, regular repairs, project monitoring (ex-ante and ex-post) costs; the costs of implementing the 2nd and subsequent stages of the structure (if a stage-based construction is planned and the stages were precisely planned)).

The costs may be calculated by making calculations for an estimate using enlarged indicators and by calculating prices according to analogical projects (with consideration to inflation).

The project benefit consist of the economy of costs listed which is received due to: reduced vehicle operating costs; due to the savings of losses associated with traffic accidents; reduced travel time costs; reduced environmental costs; reduced operational (maintenance) costs.

Based on these data a general model of investment efficiency may be formed:

$$\underset{\to 0}{S} = (S_1^{'} + S_1^{''}) - \left\{ (S_2^{'} - S_2^{''}) + (S_3^{'} - S_3^{''}) + (S_4^{'} - S_4^{''}) + (S_5^{'} - S_5^{''}) + (S_6^{'} - S_6^{''}) \right\}, \tag{1}$$

where $S_1^{'}$ – road design costs; $S_1^{''}$ – road construction costs; $S_2^{'}$ – VOC before the project implementation; $S_2^{''}$ – VOC after the project implementation; $S_3^{'}$ – AC before the project implementation; $S_3^{''}$ – AC after the project implementation; $S_4^{'}$ – TTC before the project implementation; $S_4^{''}$ – TTC after the project implementation; $S_5^{'}$ – SEE before the project implementation; $S_5^{''}$ – SEE after the project implementation; $S_6^{'}$ – RMC before the project implementation; $S_6^{''}$ – RMC after the project implementation.

Having revealed each Eq. 1 term we will receive:

$$\begin{aligned} \underset{\to 0}{S} &= (S_1^{'} + S_1^{''}) \cdot L - \Big\{ (AADT^{'} \frac{VOC^{'}}{1000} - AADT^{''} \frac{VOC^{''}}{1000}) \cdot L \cdot 365 + \\ &+ (AADT^{'} \frac{AR^{'}}{10^6} - AADT^{''} \frac{AR^{''}}{10^6}) \cdot L \cdot AC \cdot 365 + \\ &+ AADT^{'} \frac{LVi^{'}}{V^{'}} - AADT^{''} \frac{LVi^{''}}{V^{''}}) \cdot L \cdot 365 + \\ &+ \left(AADT^{'} (0{,}155 L_1 + 0{,}008 L_2) - AADT^{''} (0{,}129 L_1 + 0{,}007 L_2) \right) \cdot 365 + \\ &+ \left(D^{'} \cdot L \cdot B^{'} \cdot 1000 - D^{'} \cdot L \cdot B^{''} \right) \Big\} \end{aligned} \tag{2}$$

where: $AADT$ –annual average daily traffic, vpd; VOC – vehicle operating costs, EUR/km, where:

$$VOC = \sum_{i=1}^{m} VOC_i \cdot k_s \cdot k_{tf} \ ,$$

where: VOC_i – sum of vehicle operating costs by category of transport (car, busses, heavy trucks, and prime movers); k_s – coefficient of traffic flow distribution speed; k_{tf} – coefficient of traffic flow distribution; L – length of road section, km; L_1 – length of road section in the build-up area, km; L_2 – length of road section in the not build-up area, km; AR –accident coefficient in the section under study; AC – average costs per one accident in the road section, EUR, where:

$$AC = \sum_{i=1}^{m} AC_i \cdot n \,,$$

where: AC_i – sum of accident costs (accident with deaths, accident with injuries, technical accident); n – number of accidents; V – vehicle speed in the road section, km/h; LV_i – travel time costs, EUR/h, where:

$$LVi = \sum_{i=1}^{m} LVi_i \cdot k_{tf} \ ,$$

where: LVi_i – sum of travel time costs by transport category; k_{tf} – coefficient of traffic flow distribution; D – maintenance costs for the road, EUR/1000m^2, where:

$$D = \sum_{i=1}^{m} D_i \ ,$$

where: D_i – sum of maintenance costs (repairing, cleaning, winter maintenance, etc.), B – width of the road section, m.

3. REALIZATION OF THE MODEL

This model enables to compare the types of road construction work for all types of roads, i.e. road reconstruction, alignment of the new road, paving of the gravel road. For the practical application of the model, as an example, the paving of 1 km of the gravel road was selected.

During operation of the paved road, its technical characteristics change, therefore we take into consideration an increase in irregularities of the road surface, and its reduction in the 8th and 16th years of project existence when an overhaul of the surface is carried out [3]. Based on the correlation between the VOC (1000 vehicles km/year) and the roughness of the road surface [2, 10] the regression was deduced (Eq. (3)).

$$y = 15{,}534x + 287{,}72; \tag{3}$$

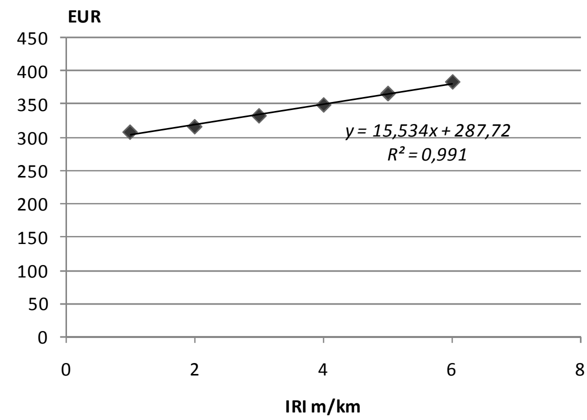

Fig. 1. VOC correlation between the surface roughness in case of the paved road
Рис. 1. Корреляция между неровностью дороги и эксплуатационными расходами автотранспортных средств на асфальтированной дороге

The surface roughness index of the paved road ranges between 1.5 to 6 m/km [1; 3] therefore in the Fig.1 the following range for deducing was used (Eq. (4)).

$$y = 20{,}922x + 254{,}52; \tag{4}$$

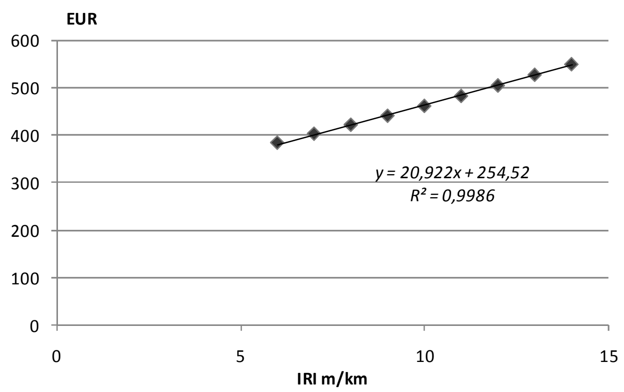

Fig. 2. VOC correlation between the surface roughness in case of the gravel road
Рис. 2. Корреляция между неровностью дороги и эксплуатационными расходами автотранспортных средств на гравийной дороге.

Based on statistics, the index of roughness the gravel road surface may reach up to 18 m/km, yet an index of 14 m/km is already considered as a very poor condition of road surface. The index range of the country's gravel surface roughness is very wide – from 5 to 14 m/km [6]. The lower limit is achieved right after grading, yet, depending on the weather conditions and traffic volume, the coefficient increases fast. Therefore in the Fig. 2 the marginal limits from 5 to 14 m/km were selected.

Vehicle speed also influencing VOC. Based on the correlation between the vehicles speed and the coefficients of traffic flow distribution speed [10] the regression was deduced (Eq. (5)).

$$y = 0{,}0003x^2 - 0{,}0394x + 2{,}3757 \quad (5)$$

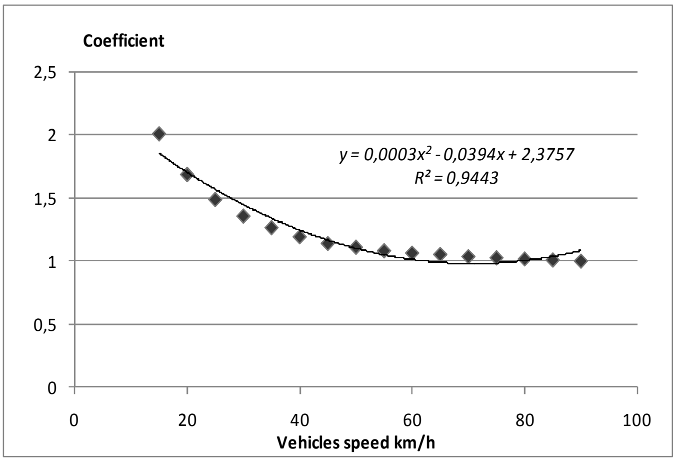

Fig. 3. Correlation between the vehicles average speed and rise in VOC
Рис. 3. Подорожание эксплуатационных расходов автотранспортных средств в зависимости от скорости

To assess the Average annual daily traffic (*AADT*), data of *Technical and economical monitoring of the gravel road paving works financed by the European Union Regional Development Funds for the 2004–2005 period* (TKTI, Kaunas, 2007) road was used. In a number of sections, the traffic volume changes insignificantly after the project implementation, yet sometimes it increases even 3 times. According to the values of 54 sections, traffic volume fluctuating from 50 to 550 vpd on the gravel roads and from 55 to 605 after paving these roads. Based on these data, a regression equation of traffic volume with respect to random sections is deduced in the Fig. (4,5) (Eq. 6,7).

$$y = 48{,}645x - 16{,}055 \quad (6)$$

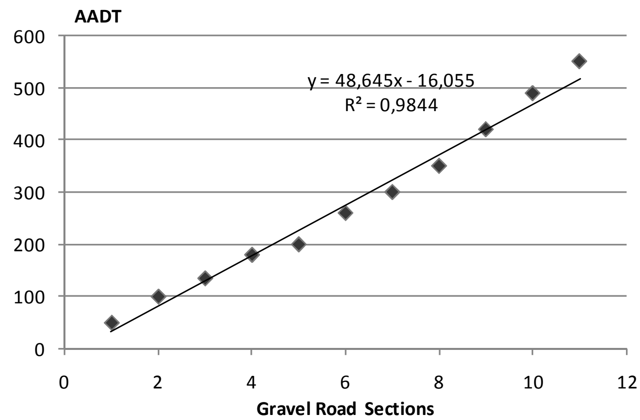

Fig. 4. AADT in gravel road sections
Рис. 4. Средняя суточная интенсивность движения транспорта на гравийной дороге

$$y = 53{,}51x - 17{,}66 \quad (7)$$

Average vehicle speed depends on roughness of road. According to calculations with HDM-4 model [9], the correlation between the average vehicle speed and roughness of road was determined in Fig. 4, 5. (Eq. 8, 9)

$$y = -4{,}6053x + 107{,}07 \quad (8)$$
$$y = -0{,}5238x^2 + 1{,}6429x + 89{,}452 \quad (9)$$

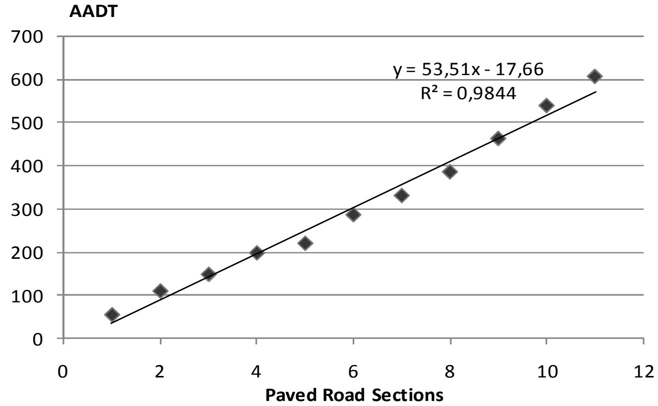

Fig. 5. AADT in road sections after paving
Рис. 5. Средняя суточная интенсивность транспортных средств после асфальтирования гравийной дороги

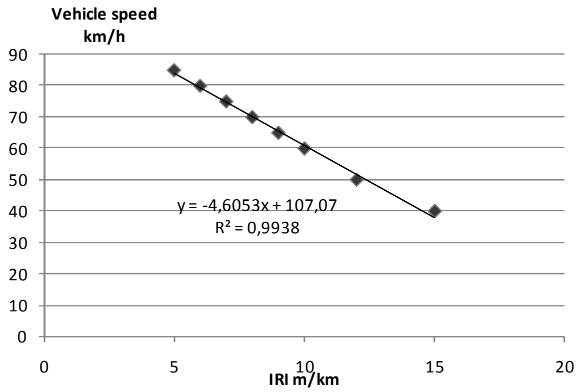

Fig. 6. Correlation between the average vehicle speed and roughness on gravel road
Рис. 6. Зависимость скорости транспортных средств от неровности на гравийной дороге

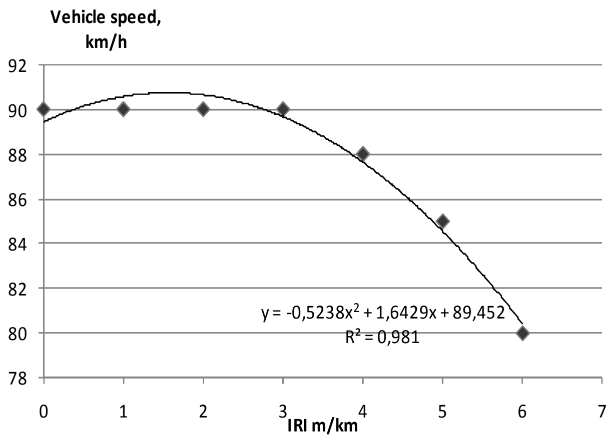

Fig. 7. Correlation between the average vehicle speed and roughness on paved road.
Рис. 7. Зависимость скорости транспортных средств от неровности на асфальтированной дороге.

According to the adaptation of the TARVAL (abbreviation from finish TARVAL: A tool for estimation of traffic safety effects of road improvement) model, developed at the Technical Research Centre of Finland (VTT) for the Finnish National Road Administration (Finnra), for Lithuania TARVAL, the rate of effect of the engineering measures on the accident rate after paving the gravel road increases from 1,0 to 1,1. Yet, after assessing the rates of integration of safe traffic measures [5], it is assumed that the accident rate decreases to 0,96 on average.

According to the statistical data of road maintenance in Lithuania for the past eight years, the costs of maintenance of the gravel road and of the paved road was calculated in Figs. 8 and 9; Eqs 10 and 11).

$$y = 14{,}052x^2 + 30{,}104x + 1038{,}7 \tag{10}$$

$$y = 5{,}621x^2 + 12{,}041x + 415{,}47 \tag{11}$$

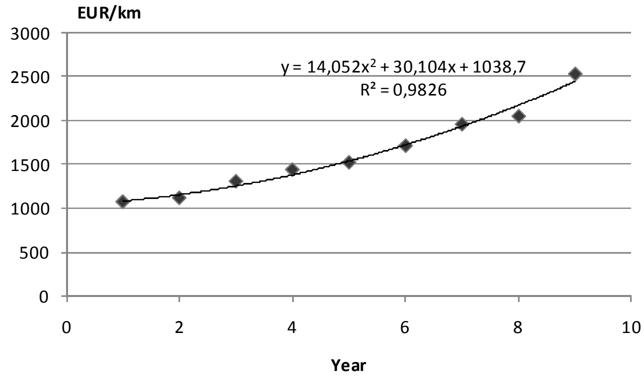

Fig. 8. RMC of the gravel road with respect to time
Рис. 8. Увеличение расходов во времени на присмотр за гравийной дорогой

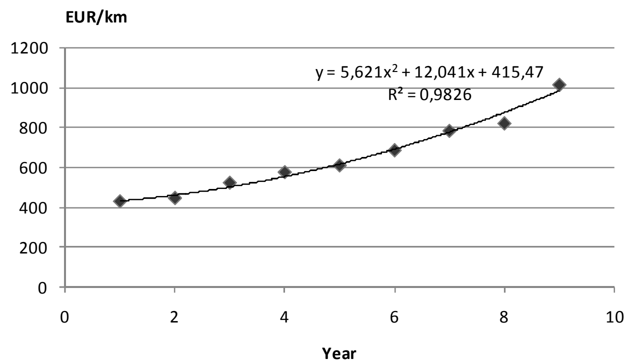

Fig. 9. RMC of the paved gravel road with respect to time
Рис. 9. Увеличение расходов во времени на присмотр за асфальтированной дорогой

Social and Environmental Effects was calculated by Lithuanian conditions.

Based on data available, the limits of change in each term of the model are expressed in regression Eq. (3-11). Having entered them into Eq. 2 and receive:

$$\begin{aligned}
\underset{\to 0}{S} = (S_1^{'} + S_1^{''}) \cdot L - \Bigg\{ & ((48{,}645x_1 - 16{,}055)\,\frac{(72{,}241x_2 + 878{,}8)(0{,}0\,003_3^2 - 0{,}0394x_3 + 2{,}3757)}{1000} - \\
& - (53{,}51x_4 - 17{,}66)\,\frac{(53{,}637x_5 + 993{,}42)(0{,}\,0003x_6^2 - 0{,}0394x_6 + 2{,}3757)}{1000}) \cdot L \cdot 365 + \\
& + ((48{,}645x_1 - 16{,}055)\,\frac{AR^{'}}{10^6} - (53{,}51x_4 - 17{,}66)\,\frac{AR^{''}}{10^6}) \cdot L \cdot AC \cdot 365 + \\
& + (48{,}645x_1 - 16{,}055)\,\frac{LVi^{'}}{(-4{,}6053x_7 + 107{,}07)} - (53{,}51x_4 - 17{,}66)\,\frac{LVi^{''}}{(-0{,}5238x_8^2 + 1{,}6429x_8 + 89{,}452)}) \cdot L \cdot 365 + \\
& + \big((48{,}645x_1 - 16{,}055)\,(0{,}535L_1 + 0{,}029L_2) - (53{,}51x_4 - 17{,}66)\,(0{,}446L_1 + 0{,}024L_2)\big) \cdot 365 + \\
& + \big((48{,}52x_9^2 + 103{,}94x_9 + 3586{,}3) - (19{,}408x_{10}^2 + 41{,}577x_{10} + 1434{,}5)\big)\Bigg\}
\end{aligned} \tag{12}$$

For the selection of optimal x_1–x_{10} values the model released using the multicriterial optimising method “LP search” [8]. Having determined the marginal values of each *x*, the method enables to select an optimal whole of all criteria. Since road parameters vary from year to year, different variables introduced each year, in the project existence cycle of 20 years. Scheme of the efficiency determination model is shown in Fig. 10.

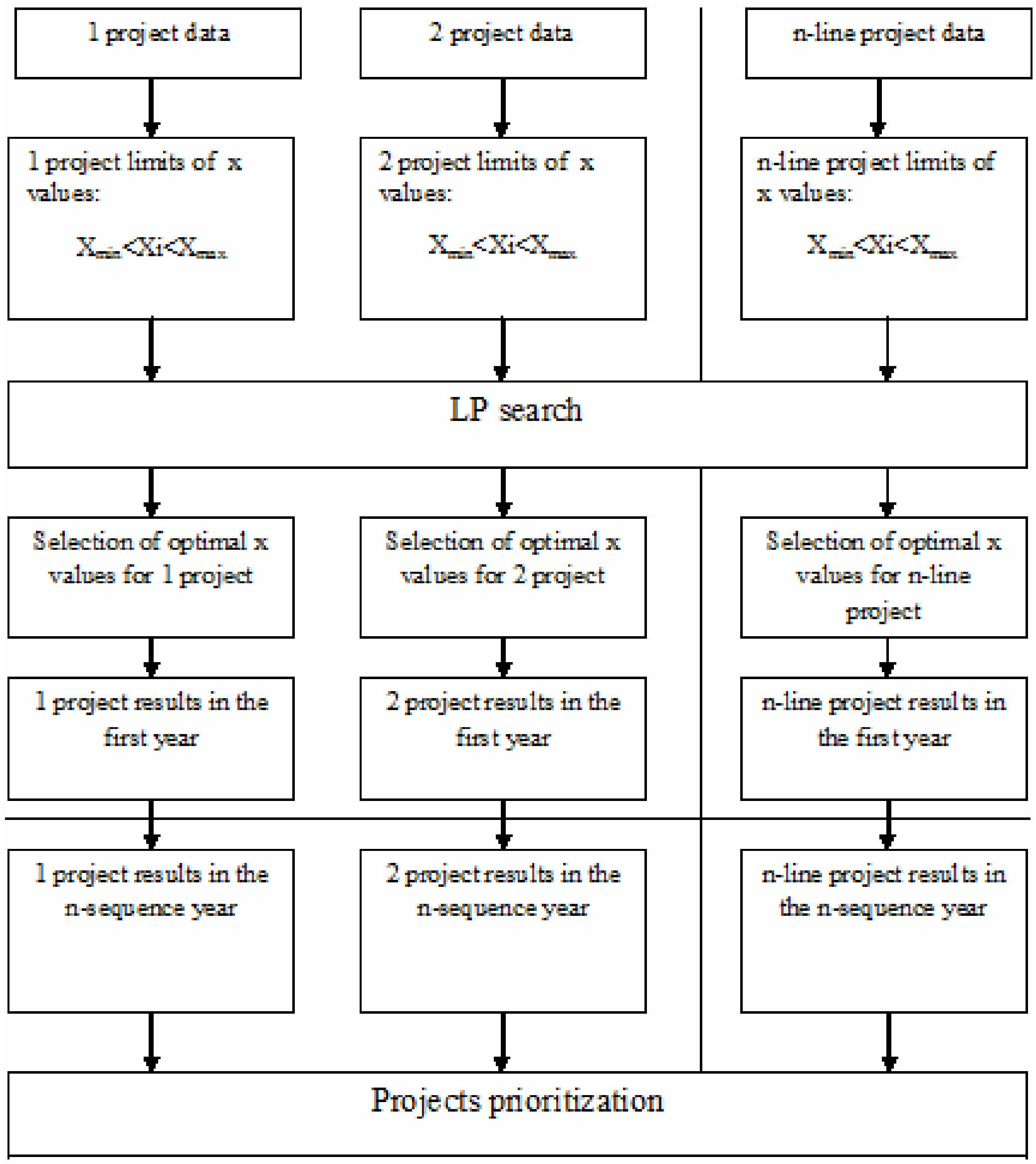

Fig. 10. Block scheme of the efficiency evaluation model
Рис. 10. Блок схема оценки эффективности проекта

Thus, having analysed the data provided a test by calculating the paving of 1 km of the gravel road with AADT 200 vpd and AADT 205–220 vpd after paving was made. According to the summaries of estimates of the 1st stage of The *Road Maintenance and Development Programme 2002–2005* and after analyzing the costs of paving the gravel roads during these 5 years, it is assumed that the design costs of 1 km to pave gravel road amounts to EUR 9745 and of the construction – to EUR 298027. Fig. 8. represents general investment payoff of paving 1 km of the gravel road, counted according to the efficiency determination model through the use of LP search.

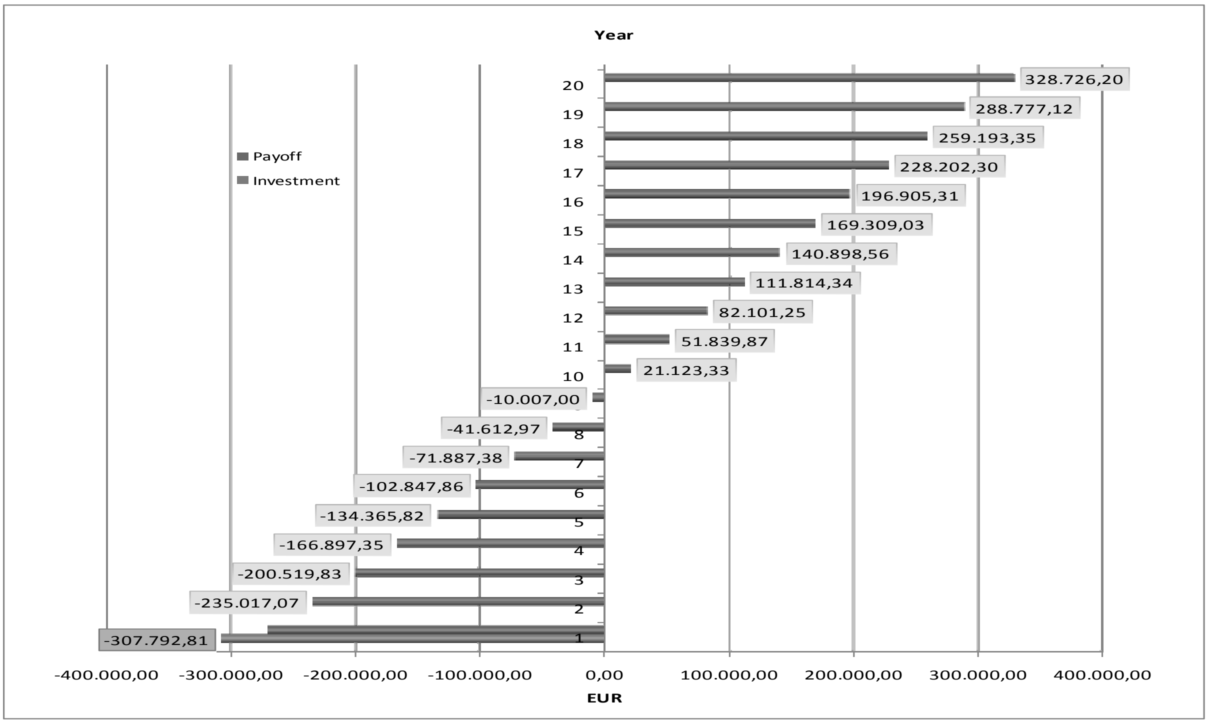

Fig. 11. General investment payoff of paving of 1 km of the gravel road
Рис. 11. Общая окупаемость инвестиций после асфальтирования одного км гравийной дороги

As it is evident, after project implementation investment pays off during 10–11th years of the project existence through the reduction of public costs, whereas at the end of the project existence 20 years cycle the benefit increases by 1.07 times in pecuniary terms.

Fig. 9. represents benefit society of paving 1 km of the gravel road, counted according to the efficiency determination model through the use of LP search.

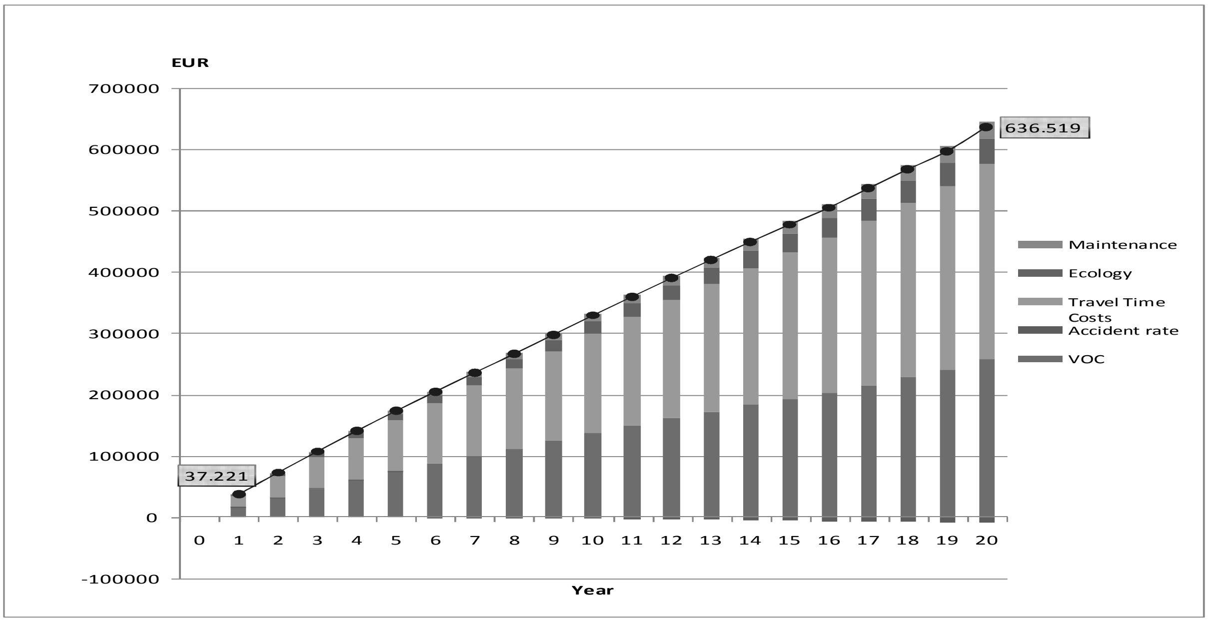

Fig. 12. Benefit society during the project 20 year existence period
Рис. 12. Общественная польза в течении 20 лет после внедрения проекта

Since an increase in the traffic volume directly affects an increase in the accident rate, the economy gradually reduces due to the accident rate, and it increases according to the arrangement of variables selected by me since the 6th year. According to statistical data, traffic volume after improving the gravel road quality by more than 50 % of the sections increases slightly, and even several times on several sections. Yet, it practically remains the same in the remaining sections. Thus, while assessing projects according to specific data, but on average values, an assumption may be made that an economy due to the accident rate would not get a negative value. In the sections where traffic volume increases several times (one of the examples, from 200 to 605 vpd), additional safety measures would be integrated (e.g. speed meters, lightening, more intensive work of traffic safety supervision officers, etc.), therefore an accident rate would become lower accordingly, thus, an assumption could be made that losses due to accident rate would not increase in these sections with high traffic intensity.

4. CONCLUSIONS

1. The calculations show that investments into the paving of 1 km of gravel after implementing project pay off during the 10–11th year of the project existence.
2. The project of gravel road pavement payoff will be most impacted by the reduction in the vehicle operating costs and in the economy of wasted time.
3. The goal of creating this is to compare the intended projects on investment into the motor roads and to identify which of them would give benefit the fastest, i.e. to prioritize them.
4. The model is open. According to the type of investment projects, or seeking to evaluate the costs in greater detail, additional terms may be entered into the model, or the unnecessary ones could be eliminated. E.g. if this is construction or reconstruction of the road, losses due to dustiness may not be taken into consideration.
5. The model could be applied as an additional measure for determining the priority of projects intended for implementation.
6. The model analyzed in the article is adapted for comparing the projects on the paving of gravel roads. It may also be adapted for other types of works (roads reconstruction, new road construction).
7. The model was realised using Lithuanian statistical data, but it could be adapted to another country.

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