

suspended monorail locomotive

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## **DYNAMIC PROCESSES DURING MONORAIL LOCOMOTIVE ROCKING AND THEIR IMPACT ON DRAW GEAR CHARACTERISTICS**

**Summary.** The article discusses the motion of the suspended monorail locomotive, interrelation between the parameters of irregularities false path and lateral rocking monorail locomotive, the values of lateral deviation for the different speeds on the monorail.

## **ДИНАМИЧЕСКИЕ ПРОЦЕССЫ РАСКАЧИВАНИЯ МОНОРЕЛЬСОВОГО ЛОКОМОТИВА И ИХ ВЛИЯНИЕ НА ПАРАМЕТРЫ ТЯГОВОГО УСТРОЙСТВА**

**Аннотация.** Рассмотрено движение подвешенного монорельсового локомотива. Установлена взаимосвязь параметров неровностей подвешенного пути и бокового раскачивания монорельсового локомотива. Определены значения бокового отхода для разных скоростей движения по монорельсу.

In actual practice when a monorail locomotive is in motion vibration of it's integrated parts is unavoidable. Motive power except effect monorail track axial motion makes small vibration of complex structure. Keeping small amplitude, vibrations can be accompanied by additional forces effecting the monorail, driving and running wheels. Forces increase leads to increased wear, track deformation and as a result safety level decreasing. It's worth to remember that occurring lateral deflections can exceed fixed norms for side clearance and restrict speed substantially.

There are many designs of monorail locomotives [1-3]. Monorail locomotive motive power can be presented as one-mass dynamic model (Fig. 1). Here for simplification motive power is presented as unsprung deck. Motion speed is taken as constant. Taking into account small running wheels track, it is taken as zero, and driven frame is examined as solid body with fore plane symmetry and point-to-point suspension system. Un-balance and hydroscopic moments of transmission and motor gyrating mass are equal to zero. Driving wheels press to monorail vertical face by force, which is in direct proportion to rigidity coefficient of pressure mechanism.

In the scheme is taken into account that by vibration in point of contact of driving wheels and monorail moment of friction occurs. The monorail in consequence of it's bending and lateral rigidity is taken as motionless by draw gear movements. Due to small weight of carriages it is not taken into account. Movement of single deck is only considered, that removes couplings action.

The following draw gear signs are imposed:  $m$  – reduced mass,  $l$  – distance from mass centre to suspension point,  $h_p$  – distance from longitudinal axle of pressure mechanism springs to suspension points,  $C_g$  – coefficient of rigidity of pressure mechanism springs.

As the main reference frame Cartesian coordinate axis  $X_o$ ,  $Y_o$  и  $Z_o$ , with start point in  $O_1$  were chosen, located in the middle of front and back carriages. By draw gear movements axis  $Z_o$  is always vertical, axis  $Y_o$  is directed to movement of linear monorail axis, and axis  $X_o$  is perpendicular to flatness, which passes axis  $Z_o$  и  $Y_o$ .

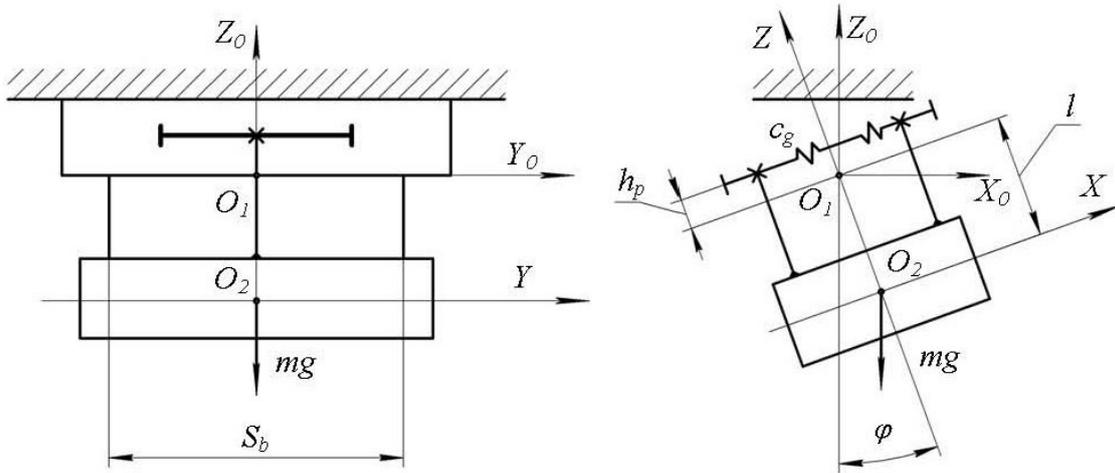


Fig. 1. Circuit of monorail locomotive  
Рис. 1. Схема монорельсового локомотива

As additional coordinates frame Cartesian coordinate axis  $X$ ,  $Y$  и  $Z$  are taken. Their start is in mass centre of moving draw gear. In static balance the main system axis  $X_o$  and  $Y_o$  are parallel to axis  $X$  and  $Y$ , and axis  $Z_o$  и  $Z$  coincide.

By monorail locomotive progressive motion system  $XYZ$  relatively to system  $X_oY_oZ_o$  makes turns of angles  $\varphi$  around the axis  $Y_o$  and turns of angles  $\gamma$  around axis  $Z_o$ , taken as reported coordinates. The reason of vibration by straight areas movements is track inequality that can be described with the help of perturbation functions in horizontal plane:

$$\Phi_1(t) = \frac{4L \sin \alpha}{\pi^2} \sum_{n=1}^n \frac{\cos(2n-1)\pi 0,5S_b/L}{(2n-1)^2} \cos(2n-1)W \left( t - \frac{\tau}{2} \right) \quad (1)$$

$$\Phi_2(t) = \frac{4L \sin \alpha}{\pi^2} \sum_{n=1}^n \frac{\sin(2n-1)\pi 0,5S_b/L}{(2n-1)^2} \sin(2n-1)W \left( t - \frac{\tau}{2} \right) \quad (2)$$

where:

$\alpha$  – axis torsion angle of related elements in horizontal plane;  $L$  – monorail section length;

$W$  – perturbation frequency,  $W = \pi V/L$ ;  $\tau$  - lag time of the second running wheels pair comparing with the first;  $n = 1, 2, 3$

Similarly perturbation functions from vertical inequality of  $\Phi_3(t)$  and  $\Phi_4(t)$  track are described, where instead of angle  $\alpha$  torsion angle in vertical plane of track  $\beta'$  is considered.

By making rolling equation we use Lagrange equation in the following form

$$\frac{d}{dt} \left( \frac{\partial T}{\partial \dot{\varphi}_i} \right) - \frac{\partial T}{\partial \varphi_i} + \frac{\partial \Pi}{\partial \varphi_i} = Q_i \quad (3)$$

where:

$T$ ,  $\Pi$  – correspondingly, kinetic and potential energies of mechanic system;

$Q_i$  – summarized force, corresponding to generalized coordinate.

Based on accepted analytical model (see Fig.1) kinetic energy is defined as sum:

$$T = \frac{I_y}{2} \dot{\varphi}^2 + \frac{m \dot{x}^2}{2} + \frac{m \dot{y}^2}{2} + \frac{m \dot{z}^2}{2} + \frac{I_z}{2} \dot{\gamma}^2 \quad (4)$$

where:

$I_y, I_z$  - correspondingly, inertia moments relatively to axis  $Y$  and axis  $Z$ .

Lateral displacement of mass center can be found:

$$x = l \sin \varphi + \Phi_1(t) \quad (5)$$

Similarly, taking into account affective perturbation from vertical inequalities  $\Phi_3(t)$ :

$$z = l - l \cos \varphi + \Phi_3(t) \quad (6)$$

Taking into account effective perturbation  $\Phi_2(t)$  turn angle of draw gear relatively to axis  $Z$  is equal to:

$$\gamma = \frac{2}{S_b} \Phi_2(t) \quad (7)$$

Derivative formulas (5), (6) and (7) using time  $t$ :

$$\dot{x} = l \cos \varphi + \dot{\Phi}_1(t) \quad (8)$$

$$\dot{z} = l \sin \varphi + \dot{\Phi}_3(t) \quad (9)$$

$$\dot{\gamma} = \frac{2}{S_b} \dot{\Phi}_2(t) \quad (10)$$

After we substitute formula (4) by square of incoming parameters, determined according to (8), (9), (10) and after making transformations we get the following:

$$T = \frac{I_y + ml^2}{2} \dot{\varphi}^2 + ml \dot{\varphi} \dot{\Phi}_1(t) \cos \varphi + ml \dot{\varphi} \dot{\Phi}_3(t) \sin \varphi + \frac{m}{2} \left( \dot{\Phi}_1^2(t) + \dot{\Phi}_3^2(t) \right) + \frac{I_z}{S_b} \dot{\Phi}_2^2(t) \quad (11)$$

We will find derivatives from formula (11) according to summarized coordinate:

$$\frac{\partial T}{\partial \varphi} = -ml \dot{\varphi} \dot{\Phi}_1(t) \sin \varphi + ml \dot{\varphi} \dot{\Phi}_3(t) \cos \varphi \quad (12)$$

$$\frac{\partial T}{\partial \dot{\varphi}} = (I_y + ml^2) \dot{\varphi} + ml \dot{\Phi}_1(t) \cos \varphi + ml \dot{\Phi}_3(t) \sin \varphi \quad (13)$$

From here:

$$\frac{d}{dt} \left( \frac{\partial T}{\partial \dot{\varphi}} \right) = (I_y + ml^2) \ddot{\varphi} + ml \ddot{\Phi}_1(t) \cos \varphi + ml \ddot{\Phi}_3(t) \sin \varphi - \quad (14)$$

$$- ml \dot{\Phi}_1(t) \dot{\varphi} \sin \varphi + ml \dot{\Phi}_3(t) \dot{\varphi} \cos \varphi$$

We will define potential energy of draw gear  $\Pi$ , deviated from vertical position on angle  $\varphi$ , as sum of potential energy  $\Pi_m$ , fitting to gravity and  $\Pi_n$  fitting to elastic force of springs. As zero position we will take stop position.

By system turn to angle  $\varphi$ , potential energy  $\Pi_m$  will be:

$$\Pi_m = mgz = mg(l - l \cos \varphi + \Phi_3(t)) \quad (15)$$

By system turn to angle  $\varphi$ , one of draw gear springs get shorter and another one get longer to  $\lambda$ , which is equal to

$$\lambda = h_p t_g \varphi \quad (16)$$

In stop position every spring is pressed with pressing force  $P_n$  to:

$$\Delta l = P_n / C_g \quad (17)$$

By system turn to angle  $\varphi$ , springs linear strain will be:

$$l_g = \Delta l + \lambda + \Phi_1(t) \cdot 1 / \cos \varphi \quad (18)$$

Correct in values of order of vanishing the formula (18) taking into account (16) can have the following form:

$$l_g = \Delta l + h_p \varphi + \Phi_1(t) \quad (19)$$

Potential energy  $\Pi_n$  will be determined as functioning of overall reaction of springs  $P_n$  by transferring of the system to zero position:

$$\Pi_n = 2 \left( \frac{C_g l_g^2}{2} - \frac{C_g \Delta l^2}{2} \right) \quad (20)$$

Taking into account (15) and (20), after transformations we will have total potential energy:

$$\begin{aligned} \Pi = & mg(l - l \cos \varphi + \Phi_3(t)) + \\ & + C_g (h_p^2 \varphi^2 + \Phi_1^2(t) + 2(\Delta l h_p \varphi + \Delta l \Phi_1(t) + \Phi_1(t) h_p \varphi)) \end{aligned} \quad (21)$$

We will find derivate with the help of summarized coordinate:

$$\frac{\partial \Pi}{\partial \varphi} = mgl \sin \varphi + 2C_g (h_p^2 \varphi + \Delta l h_p + \Phi_1(t) h_p) \quad (22)$$

According to accepted tolerances, summarized force corresponding to summarized coordinate  $\varphi$  is equal to

$$Q_\varphi = -\alpha_\varphi \dot{\varphi} \quad (23)$$

where:

$\alpha_\varphi$  - dry friction coefficient.

After we put values (12), (14), (22) and (23) in equation (3), we get:

$$\begin{aligned} (I_y + ml^2) \ddot{\varphi} + ml \cos \varphi \ddot{\Phi}_1(t) + ml \left( \ddot{\Phi}_1(t) + g \right) \sin \varphi + \\ + 2C_g (h_p^2 \varphi + \Delta l h_p + \Phi_1(t) h_p) = -\alpha_\varphi \dot{\varphi} \end{aligned} \quad (24)$$

With restriction of first order values we get small oscillation equation:

$$\begin{aligned} \ddot{\varphi} + \frac{\alpha_{\varphi} \dot{\varphi}}{I_y + ml^2} + \left( \frac{ml}{I_y + ml^2} (\Phi_3(t) + g) + \frac{2C_g h_p}{I_y + ml^2} \right) \varphi = \\ = - \frac{ml}{I_y + ml^2} \ddot{\Phi}_1(t) - \frac{2C_g h_p (\Delta l + \Phi_1(t))}{I_y + ml^2} \end{aligned} \quad (25)$$

The equation we get (25) is mathematical model of draw gear by inequality monorail movement, which differs from known ones by taking into account driving wheels, interacting with monorail vertical wall.

Equation (25) can be presented as follows:

$$\begin{aligned} \ddot{\varphi} + 2n_1 \dot{\varphi} + \left( p^2 \left( 1 + 1/g \ddot{\Phi}_3(t) \right) + h_p q_g \right) \varphi = \\ = p^2 / g \ddot{\Phi}_1(t) - q_g (\Delta l + \Phi_1(t)) \end{aligned} \quad (26)$$

where:

$$2n_1 = \frac{\alpha_{\varphi}}{I_y + ml^2};$$

$$p^2 = \frac{mgl}{I_y + ml^2};$$

$$q_g = \frac{2C_g h_p}{I_y + ml^2};$$

$$\ddot{\Phi}_1(t) = - \frac{4LW^2 \alpha}{\pi^2} \sum_{n=1}^n \cos(2n-1)\pi 0,5S_b / L \times \cos(2n-1)W(t - \tau/2);$$

$$\ddot{\Phi}_3(t) = - \frac{4LW^2 \beta}{\pi^2} \sum_{n=1}^n \cos(2n-1)\pi 0,5S_b / L \times \cos(2n-1)W(t - \tau/2).$$

In equation (26) given above, occurring forced oscillations conditional on perturbation activity from horizontal and vertical inequalities of monorail track. For solution and studying of equation we use principle of superposition, which gives us an opportunity to consider influence to lateral sway of draw gear in every plane.

If vertical inequalities are absent when  $\beta' = 0$ , equation (26) is as follows:

$$\begin{aligned} \ddot{\varphi} + 2n_1 \dot{\varphi} + (p^2 + h_p q_g) \varphi = \\ = \frac{4LW^2 p^2 \alpha}{g\pi^2} \sum_{n=1}^n \cos(2n-1)\pi 0,5S_b / L \cos(2n-1)W(t - \tau/2) - \\ - q_g \Delta l - \frac{4L\alpha q_g}{\pi^2} \sum_{n=1}^n \frac{\cos(2n-1)}{(2n-1)^2} \cos(2n-1)W(t - \tau/2). \end{aligned} \quad (27)$$

If horizontal inequalities are absent when  $\alpha = 0$ , we get:

$$\begin{aligned} \ddot{\varphi} + 2n_1 \dot{\varphi} + \left( p^2 \left( 1 - \frac{4LW^2 \beta'}{g\pi^2} \sum_{n=1}^n \cos(2n-1)\pi 0,5S_b / L \cos(2n-1)W(t - \tau/2) \right) + h_p q_g \right) \varphi = \\ = -q_g \Delta l. \end{aligned} \quad (28)$$

For the main perturbation harmonics when  $n = 1$ , equation (27) can be presented as follows:

$$\begin{aligned} \ddot{\varphi} + 2n_1 \dot{\varphi} + p_1^2 \varphi = & \frac{4LW^2 p^2 \alpha}{g\pi^2} \cos \pi 0,5S_b / L \cos W(t - \tau / 2) - \\ & - q_g \Delta l - \frac{4\alpha q_g}{\pi^2} \cos \pi 0,5S_b / L \cos W(t - \tau / 2) \end{aligned} \quad (29)$$

where:

$$p_1^2 = p^2 + q_g h_p.$$

This is nonhomogeneous equation of the second order with constant coefficients and right side unequal to zero. Full solution of equation (29) is equal to:

$$\begin{aligned} \varphi = & e^{-n_1 t} (\varphi_0 \cos p_c t + 1/p_c (\varphi_0 + n_1 \varphi_0) \sin p_c t) + \\ & + e^{-n_1 t} ((C_{01} \sin(\frac{W\tau}{2} + \varepsilon_{01}) + q_g \Delta l / p^2 - C_{03} \sin(\frac{W\tau}{2} + \varepsilon_{03})) \cos p_c t + \\ & + 1/p_c (n_1 (C_{01} \sin(\frac{W\tau}{2} + \varepsilon_{01}) + q_g \Delta l / p_1^2 - \\ & - C_{03} \sin(\frac{W\tau}{2} + \varepsilon_{03})) - W(C_{01} \cos(\frac{W\tau}{2} + \varepsilon_{01}) - \\ & - C_{03} \cos(\frac{W\tau}{2} + \varepsilon_{03}))) \sin p_c t) + C_{01} \sin(W(t - \tau / 2) - \varepsilon_{01}) - \\ & - q_g \Delta l / p_1^2 - C_{03} \sin(W(t - \tau / 2) - \varepsilon_{03}) \end{aligned} \quad (30)$$

where:

$$p_c = \sqrt{p_1^2 - n_1^2}$$

$C_{01}$ ,  $C_{03}$ ,  $\varepsilon_{01}$ ,  $\varepsilon_{03}$  - constants of integration.

Equation (30) analysis shows that summand describes own damped vibrations depending on initial conditions. With zero initial conditions the summand is equal to zero. The second summand describes own damped vibrations occurring by running-down to inequality, by steady mode act forced oscillations being described by other summands of the equation.

For other harmonics when  $n = 2, 3, 4 \dots$  or full solution of equation (27) can be presented as sum of mentioned specific solution. If we consider only steady mode the biggest lateral deviation will be equal to:

$$\begin{aligned} \varphi_y = & \frac{4LW^2 p^2 \alpha}{g\pi^2} \sum_{n=1}^n \mu_{\varphi_i} \cos(2n-1)\pi 0,5S_b / L - \\ & - q_g \Delta l / p_1^2 - \frac{4L\alpha q_g}{\pi^2} \sum_{n=1}^n \mu_{\varphi_i} \frac{\cos(2n-1)0,5S_b / L}{(2n-1)^2} \end{aligned} \quad (31)$$

where:

$\mu_{\varphi_i}$  – amplification factor of  $i$  – perturbation harmonic in general case being equal to:

$$\mu_{\varphi_i} = \frac{1}{\sqrt{(p_1^2 - (2n-1)^2 W^2)^2 + 4n_1^2 (2n-1)^2 W^2}}, (n = 1, 2, 3, \dots).$$

Equation (31) analysis we got shows that lateral deviations  $\varphi_y$  amplitude depends on some monorail track and draw gear characteristics. On Fig. 2 graph of  $\varphi_y$  against movement speed with different length of monorail sections, calculated using the following data:  $\alpha = 0,035$  rad,  $l = 0.1$  m,  $\Delta l = 0,060$  m,  $h_p = 0,063$  m,  $m = 4 \times 10^3$  kg,  $I_y = 22 \times 10^3$  kg.m<sup>2</sup>,  $n_l = 0,064$  c<sup>2</sup>. Values of incoming characteristics were taken according to characteristics of existing monorail tracks 2DMD and DMKU. From graphs follows that by movement speed increasing till 1,0...2,2 m/s, amplitude  $\varphi_y$  first increases and then decreases several times сначала with relation to the maximum. Then, with movement of less than 5 m/s speed along monorail of section length  $L=3$  m to the maximum value of  $\varphi_y=0,043$  rad corresponds speed of 1,2 m/s, and with  $L = 6$  m to the value  $\varphi_y=0.108$  rad corresponds speed of 2,2 m/s. Further increasing of section length results shift of maximum value from the working speed zone. But for curvilinear output with  $L$  value increasing twist angle between related sections  $\alpha$ , on which also depends  $\varphi_y$  amplitude. The graph is on Fig. 2.

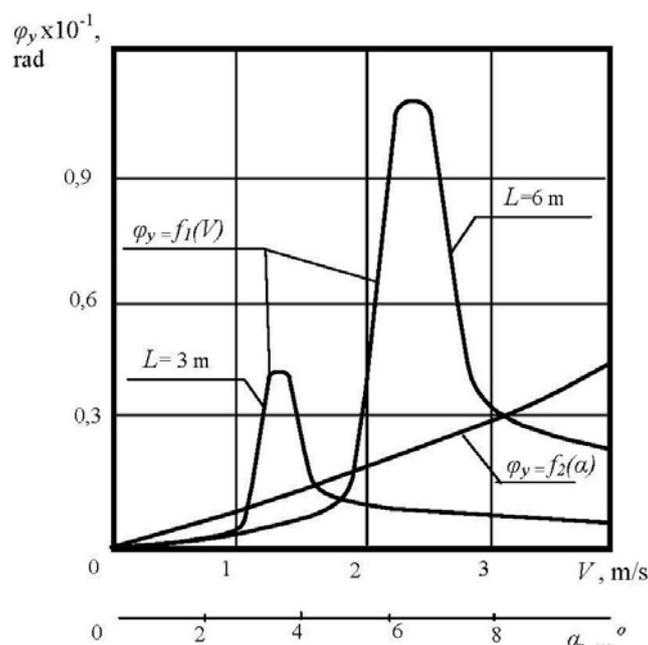


Fig. 2. Graph of  $\varphi_y$  against movement speed with different length of monorail sections

Рис. 2. График зависимости  $\varphi_y$  от скорости движения при различных длинах секций монорельса

From the graph we can see proportional correspondence between lateral deviations and twist angle with  $\alpha \leq 10^\circ$ . If angle  $\alpha$  increases twice vibration amplitude increases almost twice also. As a result increasing of monorail section length is not rational.

Continue with analysis of the influence of the draw gear's parameters. Among these are: effective dynamic mass  $m$ , stiffer modulus  $C_g$ , initial tension springs of the pressure mechanism  $\Delta l$ , axle base  $S_b$  and distance from the center of draw gear's inertia to the points of support  $l$ .

On the Fig. 3 represented characteristic of the side-sway's amplitude from the draw gear's mass, from the pressure mechanism's parameters, at a speed 3 m/s, and stretch of the monorail's section 3m.

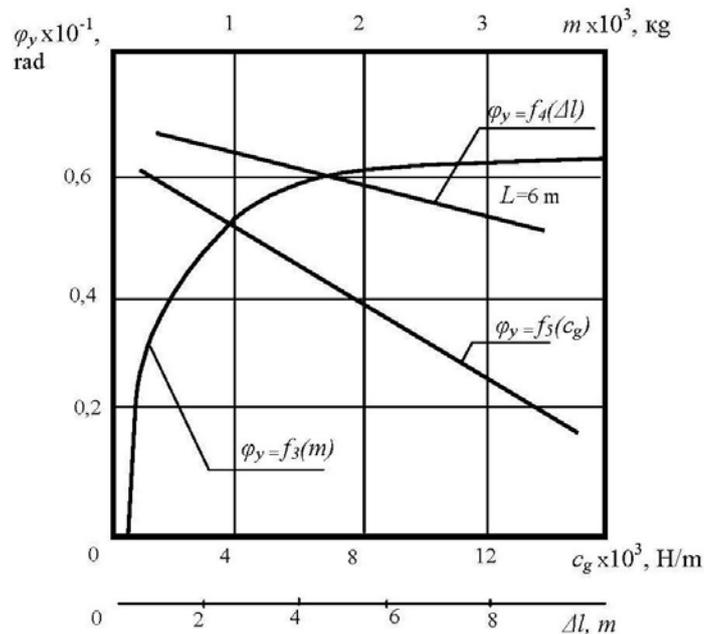


Fig. 3. Characteristic of the side-sway's amplitude from the draw gear's mass  
 Рис. 3. Зависимость амплитуды бокового раскачивания от массы

If gain  $m$  leads, first of all, to the sharp grows of the amplitude and than, under more 2500 kg, to the gradual increase of the amplitude  $\varphi_y$ , then increase of the stiffer modulus  $C_g$  and of the tension  $\Delta l$  – leads to its decrease. When  $C_g$  2 times increase, the amplitude  $\varphi_y$  decays 1,8 times and when  $\Delta l$  2 times increase – approximately by 7...8 %. Influence of the last consideration is less weighty and it has nearly no influence on  $\varphi_y$ . Inflexibility of the pressure mechanism – is one of the characteristics, changes of which can work on dimension of the drift. As it follows from the analysis, when  $C_g$  increases – the reducing of the drift is taking place all over the zone of the processes speeds.

## References

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Received 15.04.2010; accepted in revised form 27.04.2011