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## THE MEANING OF DIFFERENT LIMITERS RELATIONS FOR DEFINING PUBLIC TRANSPORT ROUTES CAPACITY

**Summary.** The mutual affecting of different limiters of capacity is a very important feature, which makes difficult the right appreciation of the transport systems. In most cases, there is no way to use procedures which define relations by correlation coefficients etc. Completely divergent character of limiters induce that only way of effective appreciation is modeling by Monte Carlo simulations.

## ZNACZENIE POWIĄZANIA RÓŻNYCH OGRANICZNIKÓW DLA OKREŚLANIA PRZEPUSTOWOŚCI TRAS KOMUNIKACYJNYCH

**Streszczenie.** Wzajemne oddziaływanie różnych ograniczników przepustowości jest bardzo ważną cechą utrudniającą właściwą ocenę układów komunikacyjnych. W większości przypadków procedury określające powiązania za pomocą współczynników korelacji nie mają zastosowania itp. Całkowicie rozbieżny charakter występujących ograniczników powoduje, że jedyną drogą prowadzącą do skutecznej oceny staje się modelowanie symulacyjne Monte Carlo.

### 1. INTRODUCTION

There are two main kinds of capacity limiters:

- windowing – for example: traffic lights – the traffic is possible only in selected time-limit window (queueing system with out of order periods) – fig. 1a,
- queueing – like stops – long time of service on selected part of track creates limits on service channels (typical queueing system) – fig. 1b.

Everyone has characteristic, different to others, pass-time distribution. Observation of mutual nearly situated limiters gives result different to expected for these limiters as independent. For example - there are, often, minimal time-distances between tram-trains determined by windowing limiter, what is an effect of nearly situated queueing limiter before. If signal which allows the traffic ST-3 (symbol taken from [2]), to pass the cross just behind the tramstop, would be administrated for about 10 seconds, it would be impossible to service this stop and pass the crossing by more than one tram at a single signal.

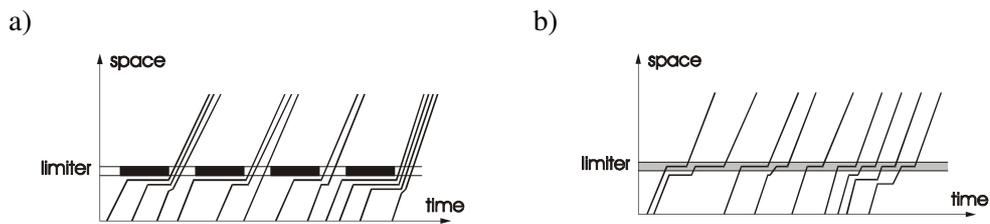


Fig. 1. Time-space diagram for capacity limiter: a) windowing; b) queueing

Rys. 1. Wykres ruchu dla ogranicznika przepustowości: a) okienkowego; b) kanałowego

They made an experiment, which has to answer the question - how large is the difference between analysis of independent and dependent limiters. There was a simulated untypical cross-roads - Central Transportation Junction in Sosnowiec. It has a small central island with a tram stop on. Traffic is directed by permanent traffic lights system. It directs the possibility of entrance and departure from the island.

Three simulations were done:

- independent processes of service probability model,
- independent processes of service Monte Carlo simulation,
- complex Monte Carlo simulation considering every mutual affecting between trams passing the cross-roads.

Subsequently the results were analyse

## 2. INDEPENDENT PROCESSES OF SERVICE PROBABILITY MODEL

Distribution of cross-roads with permanent traffic lights system passing times is shown on fig. 2. It is also defined by formulas (1) and (2). A formula (1) presents the number of trams which passes without hold to held trams ratio is the same as »green« light time to »red« light time ratio. Expected value of passing times is presented by formula (2).

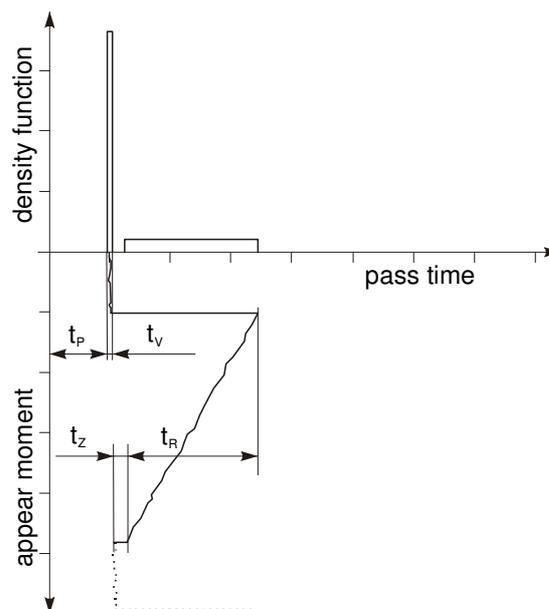


Fig. 2. Distribution of road crossing passing times with permanent traffic lights system – density function and its interpretation

Rys. 2. Rozkład czasów przejazdu przez skrzyżowanie ze stałoczasową sygnalizacją świetlną – funkcja gęstości i jej interpretacja.

$$A_{G/R} = \frac{\int_0^{t_p+t_v} F(\tau) d\tau}{\int_{t_R+t_Z+t_p+t_v}^{t_p+t_v} F(\tau) d\tau} = \frac{t_G}{t_R} \quad (1)$$

$$E(\tau) = \int_0^T F(\tau) \cdot \tau d\tau = \frac{t_G}{T} \cdot \frac{2t_p+t_v}{2} + \frac{t_R}{T} \cdot \frac{2t_p+2t_Z+2t_v+t_R}{2} \quad (2)$$

where:  $A_{G/R}$  - number of not held tram to number of held trams ratio;  $F(\tau)$  - density function of passing a part of distance in time  $\tau$ ;  $E(\tau)$  - time of passing part of distance expected value;  $t_p$  - minimal passing time without hold before passing;  $t_v$  - passing time range (connected with speed range);  $t_Z$  - time since start of showing ST-3 signal («green» light) to start the run (connected with tram inertia);  $t_G$  - «green» light time (ST-3);  $t_R$  - «red» light time (ST-1);  $T$  - traffic lights period.

Here is a simplification which consisted of:

- tram-train which is coming to cross-roads during signaled permission to pass is passing without delay,
- tram-train which is coming to cross-roads during signal „STOP” needs permanent time to restart (typical in Poland is 3÷4 s),
- tram-train is passing the cross-roads by permanent time (not large size of cross-roads (usually 7÷30 m) reduces need to apply real distribution - the presumed range 1s),
- tram-train is coming to cross-roads in completely random time, but there is no inconvenience between trams.

In reality limits like these are rare. To make the model more real, correction were made nearly permitting more than one tram. Then it is necessary to consider the mutual inconvenience. Probability of nearly appearing 2 trams is presented by formula (3). Utilizing of rectangular distribution, might be recognized as mistake, but different frequencies of service on different tram-lines is indicating it as correct.

$$P_2 = f \cdot \frac{t_R+t_Z-t_N}{2} \cdot \frac{t_R}{T} \quad (3)$$

where:  $P_n$  - probability of appearing trams in a short time (with mutual inconveniences);  $t_Z$  - time since start of showing ST-3 signal («green» light) to start the run (connected with tram inertia);  $t_N$  - sequence-time;  $t_R$  - «red» light time (ST-1);  $T$  - traffic lights period;  $f$  - averaged tram circulation frequency.

This inconvenience is defined as adding to the pass-time a sequence-time  $t_N$ .

Probability of appearing more than two trams is (with a little simplification) is presented by formula (4). In reality it is negligible.

$$P_n = P_2^{n-1} \quad (4)$$

where:  $P_n$  - probability of an appear n trams in short time (with mutual affecting).

Tramstop service time distribution is defined by measurements of real situation - more results presents [1]. Utilizing analogical formula to (3) it is included adding to service-time a holding time.



Fig. 3. Distribution of tramstop service-times – density function  
Rys. 3. Rozkład czasu obsługi przystanku – funkcja gęstości

Model is supplemented by defining formula of density function (5) which present time of passing all the cross-roads. It is founded on independence of service-time distributions of individual parts of tram net – known for example from [3].

$$F_C(\tau) = \int_0^{\tau} \int_0^{\tau-\tau_I} F_I(\tau_I) \cdot F_{II}(\tau_{II}) \cdot F_{III}(\tau - \tau_I - \tau_{II}) d\tau_{II} d\tau_I \quad (5)$$

where:  $F_C(\tau)$  - density function of passing all the cross-road in time  $\tau$ ,  $F_I(\tau)$ ,  $F_{II}(\tau)$ ,  $F_{III}(\tau)$  - density function of passing a part I, II, III of distance in time  $\tau$ .

### 3. INDEPENDENT PROCESSES OF SERVICE MONTE CARLO SIMULATION

The moments of tram-trains appear in system are stochastic process with random variable (punctuality) but it is also very important to determine the factor (timetable). It determines very much distances between trams. In different circumstances a timetable would be unnecessary. A Monte Carlo simulation allows us to introduce both the factors to notice the moment of appearance.

Using drawing, lots of start to activate the traffic lights system (in the first signal range), it is possible to precisely define if a tram appears by ST-3 («green») signal or ST-1 («red») signal and how long it will be waiting. Of course, it is also known if there is other tram held at this signal.

Analogically, it can be define as a tram stop service-time.

Independent of these processes it was utilize only to compare the results. Of course, there cannot be independence of showing a signal which acknowledges entrance to the central island of cross-roads and signal which acknowledges to leave it. It is impossible at permanent traffic lights system. The independence was acquired by drawing lots of appearing-moments to every part of the cross-roads (first traffic lights, tramstop, second traffic lights). The time of waiting and service at previous element wasn't used.

### 4. COMPLEX MONTE CARLO SIMULATION

Complex simulation utilize time of waiting at signal ST-1 to define the moment of appearing at tram stop and time of service at tramstop to define the moment of appearing at signal which approve to leave the central island. It is also checking the occupation of the central island. This island is so small that it is impossible to enter more than two tram-trains in it.

Moreover it introduces control of the occupation of the tramstop. If the service of this one is complete the tram has to wait for a signal approving to leave the central island before leaving the tramstop.

The scheme of this simulation is presented on fig. 4.



Fig. 4. Complex Monte Carlo simulation model scheme  
Rys. 4. Schemat modelu złożonej symulacji Monte Carlo

## 5. COMPARE RESULTS

The result of done analysis was defined as a passing cross-roads time distribution. Table 1 presents main parameters of distributions acquired in three simulations.

Table 1

Time of passing the cross-roads distribution parameters

Model	Independent processes of service probability model	Independent processes of service Monte Carlo simulation	Complex Monte Carlo simulation
expected value [s]	132	118	119
standard deviation [s]	41	44	64
minimum value [s]	24	24	24
maximum value [s]	308	237	472
range [s]	284	213	448

Values of these parameters are different for different models. But that is only a few values. It does not show if simulations after correction would give better results. To answer this question it is shown on fig. 5 the density functions acquired in these three simulation.

Included diagrams present that simulations of independent elements of tram-net are not suitable – properties of entire part of net is completely deformed.

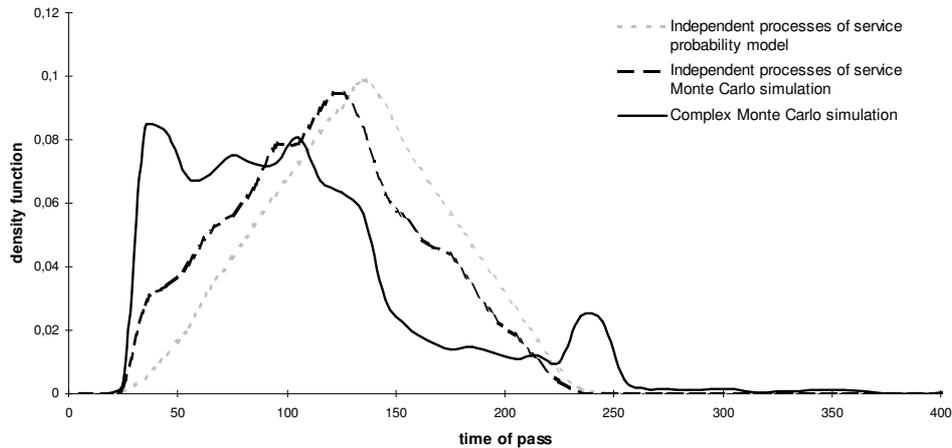


Fig. 5. Tram's passing times of Central Transportation Junction in Sosnowiec - frequency distribution functions acquired by different models

Rys. 5. Funkcje gęstości rozkładów prawdopodobieństwa czasów przejazdu tramwajów przez Centralny Węzeł Komunikacyjny w Sosnowcu, uzyskane za pomocą różnych modeli

## 6. CONCLUSION

Showed analysis proved that using, in any way, as simulation of traffic on tram net, simulation of independent elements limiting traffic flexibility and capacity is a mistake.

Projecting transport systems and giving an offer for passengers based on results of these simulations might be a failure.

Even more improperly is define the projected transport solutions based only on coefficients assigned to individual elements of tram net.

## Literature

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